CONVENTIONS FOR SPHERICAL COORDINATES

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1 The Problem

Nearly everybody uses r and θ to denote polar coordinates. Most American calculus texts also utilize θ in spherical coordinates for the angle in the equatorial plane (the *azimuth* or *longitude*), ϕ for the angle from the positive z-axis (the *zenith* or *colatitude*), and ρ for the radial coordinate. Virtually all other scientists and engineers — as well as mathematicians in many other countries — reverse the roles of θ and ϕ (and use some other letter, such as R, for the radial coordinate).

Why is this a problem? After all, the change in notation only affects students in particular fields, such as physics or electrical engineering. Furthermore, it's just a convention; surely these students have the maturity to deal with it. Based on our experience trying to implement this change during a second-year course in multivariable calculus, we feel that such sentiments underestimate the extent of the problem. Students find the complete interchange of the roles of θ and ϕ to be terribly confusing — and once confused, always confused.

Using different names for the radial coordinate, on the other hand, causes few problems. The use of r for the spherical radial coordinate can be confused with the radial coordinate in polar or cylindrical coordinates, but computations requiring both at the same time are rare. While ρ is not available to the physicist, as it is used to represent charge or mass density, students do not appear to be confused by the use of several different names for the spherical radial coordinate.

There is however a much more serious problem. Several of the most commonly used calculus texts list spherical coordinates in the order (ρ, θ, ϕ); the rest use (ρ, ϕ, θ). The first of these is left-handed! An orthogonal coordinate system is right-handed if the cross product of the first two coordinate directions points in the third coordinate direction. This is immaterial in the traditional mathematics treatment of vector calculus, but crucial to the way physicists and engineers treat the same material. These scientists often introduce basis vectors in the coordinate directions, analogous to $\{\hat{\boldsymbol{\imath}}, \hat{\boldsymbol{\jmath}}, \hat{\boldsymbol{k}}\}$ for rectangular coordinates, and it is essential that these vectors form a right-handed system. This requires that the zenith be listed before the azimuth; with the standard mathematics convention, this is (ρ, ϕ, θ). Books which use the standard mathematics definitions of the angles but write (ρ, θ, ϕ) are doing their students a major disservice, although we reiterate that this is only an issue for material covered in subsequent courses.

Some conventional choices are written in stone; others can be changed. An example of the former is the convention for the sign of the electric charge, which was adopted prior to learning that the particle which carries electric current through a wire, the electron, has negative charge. Thus, the actual motion of charged particles in a wire is in the opposite direction from the current. While this is unfortunate, everybody agrees on this choice; it would be foolish to try to change it now. When not everyone agrees, however, someone must yield. Perhaps the most famous example of this was in 1967, when Sweden changed overnight from driving on the left to driving on the right!

We argue that the conflict between the different conventions for spherical coordinates should be dealt with in the same manner: overnight conversion to a single standard. But what standard should that be?

2 Proposal

There is a uniform standard for the use of spherical coordinates in applications, which is nowhere more apparent than in the definition of spherical harmonics. These special functions on the sphere are widely used, notably in the quantum mechanical description of electron orbitals, which in turn underlies much of chemistry. It can not be stated too strongly that *everyone* writes the spherical harmonics as $Y_{\ell m}(\theta, \phi)$, where θ is the zenith and ϕ the azimuth. There is simply no way to change this convention, which is embedded in generations of standard reference books.

We propose that these conventions be adopted by mathematicians.

There are two basic objections to this. The first is that essentially all calculus texts use the other convention. The resolution of this problem is simple, at least in theory: Change the textbooks. However painful a process this is, it could be accomplished within a few years — roughly the lifetime of an edition of a calculus textbook. By contrast, changing the conventions in spherical harmonics would be virtually impossible — the lifetime of reference books far exceeds that of textbooks. Furthermore, many calculus texts must already make a nontrivial change in order to correct the problem with left-handed coordinates.

The other objection is at first sight more problematic: It is confusing to use the same label, θ, for two different angles in polar and spherical coordinates. This objection can also be easily resolved, even if the resolution may not be popular: Change the conventions for polar coordinates, that is, use ϕ rather than θ. This will be a painful change for many of us, but it could be accomplished rather quickly by changing the textbooks.

This proposal is not original with us. It is in fact the standard usage in numerous other countries, it is used in some textbooks in electrical engineering, and it is starting to be used in some physics textbooks. But the strongest argument in favor of such a change is the number of students it would affect. Essentially all current physics and electrical engineering students in the US must already make this change during the course of their education. Worse yet, this process repeats itself every year. Surely it would make more sense for the much smaller number of faculty currently teaching this material to make the change. Once.

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