Bridging the GAP between Mathematics and the Physical Sciences

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http://www.math.oregonstate.edu/bridge
Support

• Mathematical Association of America
  – Professional Enhancement Program

• Oregon State University
  – Department of Mathematics
  – Department of Physics

• National Science Foundation
  – DUE-9653250  – DUE-0231032
  – DUE-0088901  – DUE-0231194
  – DUE-0618877
Support

- Grinnell College – Noyce Visiting Professorship
- Mount Holyoke College – Hutchcroft Fund
- Oregon Collaborative for Excellence in the Preparation of Teachers
The Grand Canyon
What Are Functions?

\[ T(x, y) = k(x^2 + y^2) \]

\[ T(r, \theta) = ? \]
What Are Functions?

Suppose the temperature on a rectangular slab of metal is given by:

\[ T(x, y) = k(x^2 + y^2) \]

Physics: \[ T(r, \theta) = k r^2 \]

Math: \[ T(r, \theta) = k(r^2 + \theta^2) \]
What Are Functions?

**Math:**

\[ T = f(x, y) = k(x^2 + y^2) \]
\[ T = g(r, \theta) = k r^2 \]

**Physics:**

\[ T = T(x, y) = k(x^2 + y^2) \]
\[ T = T(r, \theta) = k r^2 \]
What’s Wrong?

\[ y = x^2 \]

**Challenge**: Find physical context.

A square has length 5 and area 25.

You just won 5 in the lottery.
Dimensions and Units

- **Dimensions**
  - Length
  - Area
  - Temperature
  - Time

- **Units**
  - cm, km, mi, ft, in
  - cm², km², acres
  - K, ºF, ºC
  - sec, hr, min
Angles

• Angles are *dimensionless*!
  – The *units* are degrees (or radians).
  – $90^\circ$ really means “$\frac{1}{4}$ of the way around”.
  – In radians, $s=r\theta$. 
Area
Perimeter?
Interactive Classroom
Group Activities

• Task Master
  Keeps group on track:
  “What you had for lunch doesn’t seem relevant.”

• Cynic
  Questions everything:

• Recorder
• Reporter
What Are Vectors?

Mathematics:
• Triples of numbers

\[ \vec{v} = \langle v_1, v_2, v_3 \rangle \]

Physics:
• Arrows in space

\[ \vec{v} = v_1 \hat{i} + v_2 \hat{j} + v_3 \hat{k} \]
1. In the small town of Coriander, the library can be found by starting at the center of the town square, walking 25 meters north, turning 90° to the right, and walking a further 60 meters.

2. Magnetic north in Coriander is approximately 14° east of true north. If you use a compass to find the library (!), the above directions will fail. Instead, you must walk 39 meters in the direction of magnetic north, turn 90° to the right, and walk a further 52 meters.
Coriander

\[ \text{Diagram 1:} \quad y = 25, \quad x = 60 \]

\[ \text{Diagram 2:} \quad y = 39, \quad x = 52 \]
Coriander
Where on Earth is Coriander?
MAA Resources

• CUPM
  (MAA Committee on the Undergraduate Program in Mathematics)
  – Curriculum Guide

• CRAFTY
  (Subcommittee on Curriculum Renewal Across the First Two Years)
  – Voices of the Partner Disciplines
    http://www.maa.org/cupm/crafty
Flux is the total amount of electric field through a given area.

\[ \Phi = \sum \mathbf{E} \cdot d\mathbf{a} \]

- Perpendicular Component
- Normal Vector

Over all rectangles
Visualization—Level Curves

\[ f(x,y) = 50 - x^2 - y^2 \]
Visualization—Graph

\[ f(x,y) = 50 - x^2 - y^2 \]
\[ \vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 + u_3 v_3 \]

\[ \Rightarrow (\vec{u} - \vec{v}) \cdot (\vec{u} - \vec{v}) = \vec{u} \cdot \vec{u} + \vec{v} \cdot \vec{v} - 2 \vec{u} \cdot \vec{v} \]

\[ |\vec{u} - \vec{v}|^2 = |\vec{u}|^2 + |\vec{v}|^2 - 2 |\vec{u}| |\vec{v}| \cos \theta \]

(Law of Cosines)
Start with Geometry!

\[ (\mathbf{u} + \mathbf{v}) \cdot \mathbf{w} = \mathbf{u} \cdot \mathbf{w} + \mathbf{v} \cdot \mathbf{w} \]

\[ \Rightarrow (u_1 \mathbf{i} + u_2 \mathbf{j}) \cdot (v_1 \mathbf{i} + v_2 \mathbf{j}) = u_1 v_1 + u_2 v_2 \]

(get Law of Cosines for free!)
Use both!

\[ \vec{u} = \cos(\alpha) \hat{i} + \sin(\alpha) \hat{j} \]
\[ \vec{v} = \cos(\beta) \hat{i} + \sin(\beta) \hat{j} \]
\[ \vec{u} \cdot \vec{v} = \cos(\alpha - \beta) \]

(get addition formulas for free!)
The Bridge Project

- Small group activities
- Instructor’s Guide
- Study Guide
- CWU, LBCC, MHC, OSU, UPS, UWEC
- Workshops

http://www.math.oregonstate.edu/bridge
http://www.physics.oregonstate.edu/bridge
The Bridge Project

- **Differentials**
  - Use what you know!
- **Multiple Representations**
- **Symmetry**
  - Curvilinear coordinates and adapted bases
- **Geometry**
  - Vectors; div, grad, curl; …
Differentials

• Substitution

\[ \int 2x \sin x \, dx \]
\[ u = x^2 \]
\[ du = 2x \, dx \]

• Chain Rule

\[ x = \cos \theta \]
\[ dx = -\sin \theta \, d\theta \]
\[ u = x^2 \]
\[ du = 2x \, dx \]
\[ = 2 \cos \theta (-\sin \theta \, d\theta) \]
Vector Differentials
Vector Differentials

\[ d\vec{r} = dx \hat{i} + dy \hat{j} \]

\[ d\vec{r} = dr \hat{r} + r \, d\phi \hat{\phi} \]
Vector Differentials

- **Line Integrals**
  \[ \int 
  \vec{F} \cdot d\vec{r} \]
  \[ \int f \, ds \quad ds = \left| d\vec{r} \right| \]

- **Surface Integrals**
  \[ \int \int 
  \vec{F} \cdot d\vec{S} \quad d\vec{S} = d\vec{r}_1 \times d\vec{r}_2 \]
  \[ \int \int f \, dS \quad dS = \left| d\vec{r}_1 \times d\vec{r}_2 \right| \]

- **Volume Integrals**
  \[ dV = (d\vec{r}_1 \times d\vec{r}_2) \cdot d\vec{r}_3 \]

- **Gradient**
  \[ df = \nabla f \cdot d\vec{r} \]
Central Oregon Consortium (MSP)

- OR Dept of Ed
- 4 rural school districts + Oregon State U
- Two–week summer institute
- On-site classes (at least 8 hrs each month)
- Online discussion
- Observations
Goals

1. Significantly increase the mathematics achievement of students.
2. Significantly increase the percentage of highly qualified teachers.
3. Identify a professional development program to meet the needs of teachers.
4. Build district leadership to sustain project effects.
Walking the Dog

Your friend is doing a study of how much people exercise in their normal daily activities. Part of this study involves making graphical representations of distances and speeds of typical activities. You volunteer to help, using a perfect example from the previous weekend:
Walking the Dog

You went for a walk with your Great Dane early on a beautiful Saturday morning. For the first five minutes, you strolled along together peacefully, 3 blocks down the road, until the dog found a fire hydrant and insisted on stopping for a minute. You continued to stroll along again for another two blocks when a squirrel jumped out in front of you and led you on a merry chase: for two minutes you went forward and back, forward and back about a block each time, before the squirrel finally ran up a tree. It took you another three minutes before you could persuade the dog to leave the tree. By that time, you were late for lunch and hurried back home arriving breathless 20 minutes from the time that you set out.
Walking the Dog

- 5 minutes, 3½ blocks
- 1 minute, fire hydrant
- 2 blocks
- 2 minutes, squirrel
- 3 minutes, tree
- hurry home
- 20 minutes total
Walking the Dog

Distance (in blocks)

Time (in minutes)
Walking the Dog

![Graph showing distance vs time for walking the dog.]

- Time (in minutes): 0, 5, 6, 9, 11, 14, 20
- Distance (in blocks): 0, 3.5, 7, 10.5, 14, 17.5, 21

Points labeled A, B, C, D, E, F on the graph.
Context Rich Problems

Just as you turn onto the main avenue from a side street with a stop sign, a city bus going 30-mph passes you in the adjacent lane. You want to get ahead of the bus before the next stoplight which is two blocks away. Each block is 200-ft long and the side streets are 25-ft wide, while the main avenue is 60-ft wide. If you increase your speed at a rate of 5-mph each second, will you make it? (No Picture)

Patricia Heller and Kenneth Heller, University of Minnesota
http://groups.physics.umn.edu/physed/Research/CRP/crintro.html
Oregon Mathematics Leadership Institute (OMLI)

• NSF-funded partnership project—OSU/PSU/TDG/10 OR school districts (NSF/EHR–0412553; ODE/Oregon ESEA Title II-B MSP)

• Aimed at increasing mathematics achievement of K–12 students

• 3-week intensive summer institutes in 6 different mathematics content areas and in leadership skills
Course goals and objectives:

- Develop geometry content knowledge in K–12 teachers
  - Parallel and perpendicular lines, midpoints, perpendicular bisectors, circles
  - Squares, triangles, perimeter, tessellations
- Allow K–12 teachers to make connections between Euclidean and non-Euclidean geometries by examining similarities and differences
The Geometry Team

- One faculty member from OSU with geometry expertise and interests in mathematics education (planning);
- One faculty member from WOU with experience teaching in-service and pre-service K–12 teachers;
- Two instructors from OSU with varied teaching experiences;
- One master teacher, currently working with pre-service teachers at Univ. of Portland.
Mathematics $\neq$ Science

- Science is about things.
- Scientists can’t change the problem.
Simple Curriculum Additions

• Equations describe the relationships between physical quantities.
• Integrals involve chopping a part of space and adding up a physical quantity on each piece.
• Limits (and approximations) require one to say what dimensionless quantity is small.
Summary

Geometric visualization is the key to bridging the gap between mathematics and the physical sciences.
A Radical View of Calculus

- The central idea of calculus is not the limit.
- The central idea of derivatives is not slope.
- The central idea of integrals is not area.
- The central idea of curves and surfaces is not parameterization.
- The central representation of a function is not its graph.
A Radical View of Calculus

- The central idea of calculus is the differential.
- The central idea of derivatives is rate of change.
- The central idea of integrals is total amount.
- The central idea of curves and surfaces is “use what you know”.
- The central representation of a function is data attached to the domain.