Support

• National Science Foundation
  – DUE-0088901
• Oregon Collaboration for Excellence in the Preparation of Teachers
• Oregon State University
  – L. L. Stewart
  – Honors College
  – Research Office
• Mount Holyoke College
  – Hutchcroft Fund
Why Study Vector Calculus?

Mathematics:
• Visualization
• Geometric Reasoning
• Multiple Representations
• Reinforce Calculus Ideas
• Differential Geometry
• Complex Variables

Physics:
• Visualization
• Geometric Reasoning
• Multiple Representations
• Symmetry
• Electromagnetism!!
• Classical Mechanics
What Are Vectors?

Mathematics:
• Triples of numbers

\[ \vec{v} = \langle v_1, v_2, v_3 \rangle \]

Physics:
• Arrows in space

\[ \vec{v} = v_1 \hat{i} + v_2 \hat{j} + v_3 \hat{k} \]
Do You Do This?

\[ \vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 + u_3 v_3 \]

\[ |\vec{u}| = \sqrt{u_1^2 + u_2^2 + u_3^2} \]

Or This?

\[ \vec{u} \cdot \vec{v} = |\vec{u}| |\vec{v}| \cos \theta \]

\[ |\vec{u}| = \sqrt{\vec{u} \cdot \vec{u}} \]
Group Activities

• Task Master
  Keeps group on track:
  “What you had for lunch doesn’t seem relevant.”

• Cynic
  Questions everything:

• Recorder
• Reporter
Group Activities
Sample Group Activity

Find the angle between a diagonal of a cube and an edge
Group Activity

• Emphasizes that vectors are arrows
• Combines geometry and algebra
• Uses multiple representations

\[
\begin{align*}
geometry: \quad \vec{u} \cdot \vec{v} &= |\vec{u}| |\vec{v}| \cos \theta \\
\text{algebra:} \quad \vec{u} \cdot \vec{v} &= (u_1 \hat{i} + u_2 \hat{j} + u_3 \hat{k}) \cdot (v_1 \hat{i} + v_2 \hat{j} + v_3 \hat{k}) \\
\text{memory:} \quad \vec{u} \cdot \vec{v} &= u_1 v_1 + u_2 v_2 + u_3 v_3
\end{align*}
\]
Student Quotes

I can't believe I thought it was possible to succeed in this class without doing the labs!

(This student chose not to do the labs, dropped out, then retook the course.)

The concepts are simple and powerful. You just have to think.

Excellent! Five stars for learning and head-scratching value.

Studying in a group is so important.

Learned how to do problems in more than one way.

Took awhile, but started to make sense at the end. (All of it.) Smile.

I just want to mention that this lab section has been very valuable to my understanding of the material. Although I initially felt it would be "just another thing to get done", I now see it to be a very beneficial component to this course.

I was very surprised that I knew more than I thought.

Surprised by how much we had to recall from last week. This is a good thing.

What I want to know is... Why does it always seem so obvious after the fact?

I learned that math was all interlinked. I can list a whole lot of methods and theorems that all use different ideas, but the math ends up all the same.
Differentials

\[ df = \frac{df}{dx} \]

Substitution

\[ \int 2x \sin x \, dx \]

\[ u = x^2 \]

\[ du = 2x \, dx \]

Chain Rule

\[ x = \cos \theta \]

\[ dx = -\sin \theta \, d\theta \]

\[ du = 2x \, dx = 2\cos \theta(\sin \theta \, d\theta) \]
Vector Differentials

\[ d\vec{r} = dx \hat{i} + dy \hat{j} \]

\[ d\vec{r} = dr \hat{r} + r \, d\theta \hat{\theta} \]
The Bridging Project

• Differentials (Use what you know!)
• Exploiting multiple representations
• Symmetry (adapted bases, coordinates)
• Geometry (vectors, div, grad, curl)
• Computer visualization (Maple, Excel)
• Active engagement (groups, flashcards)
Early Problems

• The “gap” is bigger than expected
• Difficulty relating labs to lectures
• Insufficient emphasis on homework
• Open-ended problems make planning hard
• Some students don’t like small groups
• Keeping groups on track
Early Successes

• Student enthusiasm – many lightbulbs!
• Students use geometric reasoning
• Conceptual understanding appears better
• Some at-risk students blossom
• Student camaraderie
Current Status

• Proof of Concept (2001/2002)
  – Labs developed and tried
  – Instructors’ Guide in progress

• “Alpha” testing (2002)
  – Other instructors (OSU/MHC)

• Beta testing (2003 & beyond)
  – Other institutions (contact us if interested!)