Bridging the GAP between Mathematics and Physics

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http://www.physics.oregonstate.edu/bridge
Support

• **Mathematical Association of America**
  – Professional Enhancement Program

• **Oregon State University**
  – Department of Mathematics
  – Department of Physics

• **National Science Foundation**
  – DUE-9653250  – DUE-0231032
  – DUE-0088901  – DUE-0231194
Support

- Grinnell College
  - Noyce Visiting Professorship
- Mount Holyoke College
  - Hutchcroft Fund
- Oregon Collaborative for Excellence in the Preparation of Teachers
The Grand Canyon
What Are Functions?

\[ T(x, y) = k(x^2 + y^2) \]

\[ T(r, \theta) = ? \]
What Are Functions?

Suppose the temperature on a rectangular slab of metal is given by:

\[ T(x, y) = k(x^2 + y^2) \]

Physics: \[ T(r, \theta) = k r^2 \]

Math: \[ T(r, \theta) = k(r^2 + \theta^2) \]
What Are Functions?

Math:

\[ T = f(x, y) = k(x^2 + y^2) \]
\[ T = g(r, \theta) = kr^2 \]

Physics:

\[ T = T(x, y) = k(x^2 + y^2) \]
\[ T = T(r, \theta) = kr^2 \]
Mathematics ≠ Physics

- Physics is about things.
- Physicists can’t change the problem.
Physics is about things

• What sort of a beast is it?
  – Scalar fields, units, coordinates, time.

• Physics is independent of coordinates.
  – Vectors as arrows, geometry of dot and cross.

• Graphs are about relationships of physical things.
  – Fundamental physics is 3 dimensional.
  – 3-d graphs of functions of 2-variables are misleading.
  – Hills are not a good example of functions of 2 variables.
  – Use of color.

• Fundamental physics is highly symmetric.
  – Spheres and cylinders vs. parabolas.
  – Interesting physics problems can involve trivial math.
  – Adapted basis vectors, such as \( \hat{r}, \hat{\theta}, \hat{\phi} \).
Physicists can’t change the problem.

- Physics involves creative synthesis of multiple ideas.
- Physics problems not nec. well-defined math problems.
  - No preferred coordinates or independent variables.
  - No parameterization.
  - Unknowns don’t have names.
  - Getting to a well-defined math problem is part of the problem.
  - If you can’t add units, it’s a poor physics problem.
- Physics problems don’t fit templates.
  - Template problem-solving vs. skills.
  - A few key ideas are remembered best later.
- Physics involves interplay of multiple representations.
  - Dot product.
Differentials

• Substitution

\[ \int 2x \sin x \, dx \]
\[ u = x^2 \]
\[ du = 2x \, dx \]

• Chain Rule

\[ x = \cos \theta \]
\[ dx = -\sin \theta \, d\theta \]
\[ u = x^2 \]
\[ du = 2x \, dx \]
\[ = 2 \cos \theta (-\sin \theta \, d\theta) \]
Vector Differentials

\[ d\vec{r} = dx \hat{i} + dy \hat{j} \]

\[ d\vec{r} = dr \hat{r} + r \, d\phi \hat{\phi} \]
Spherical Coordinates

\( q: \) Radius
\( \alpha: \) Zenith (colatitude)
\( \beta: \) Azimuth (longitude)

Math: \((\rho, \phi, \theta)\)

Physics: \((r, \theta, \phi)\)
Vector Differentials

- **Line Integrals**
  \[ \int \vec{F} \cdot d\vec{r} \]
  \[ \int f \, ds \quad ds = \left| d\vec{r} \right| \]

- **Surface Integrals**
  \[ \iint \vec{F} \cdot d\vec{S} \quad d\vec{S} = d\vec{r}_1 \times d\vec{r}_2 \]
  \[ \iint f \, dS \quad dS = \left| d\vec{r}_1 \times d\vec{r}_2 \right| \]

- **Volume Integrals**
  \[ dV = (d\vec{r}_1 \times d\vec{r}_2) \cdot d\vec{r}_3 \]

- **Gradient**
  \[ df = \nabla f \cdot d\vec{r} \]
In a Nutshell

*Geometric visualization is the key to bridging the gap between mathematics and physics.*
Context Rich Problems

Just as you turn onto the main avenue from a side street with a stop sign, a city bus going 30-mph passes you in the adjacent lane. You want to get ahead of the bus before the next stoplight which is two blocks away. Each block is 200-ft long and the side streets are 25-ft wide, while the main avenue is 60-ft wide. If you increase your speed at a rate of 5-mph each second, will you make it? (No Picture)

Patricia Heller and Kenneth Heller, University of Minnesota
Interactive Classroom
Lecture vs. Activities

• The Instructor:
  – Paints big picture.
  – Inspires.
  – Covers lots fast.
  – Models speaking.
  – Models problem-solving.
  – Controls questions.
  – Makes connections.

• The Students:
  – Focus on subtleties.
  – Experience delight.
  – Slow, but in depth.
  – Practice speaking.
  – Practice problem-solving.
  – Control questions.
  – Make connections.
Socratic vs. Groups

How does it feel to teach in these ways?

\[ \int_{\text{class}} d\, \text{knowledge} \quad \text{vs.} \quad \int_{\text{class}} d\, \text{questions} \]

Everyone knows everything vs. No one knows anything
Group Activities

• Task Master
  Keeps group on track:
  “What you had for lunch doesn’t seem relevant.”

• Cynic
  Questions everything:

• Recorder
• Reporter
Flux is the total amount of electric field through a given area.

\[ \Phi = \sum \overrightarrow{E} \cdot d\overrightarrow{a} \]
Do You Do This?

\[ \vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 + u_3 v_3 \]

\[ |\vec{u}| = \sqrt{u_1^2 + u_2^2 + u_3^2} \]

Or This?

\[ \vec{u} \cdot \vec{v} = |\vec{u}| |\vec{v}| \cos \theta \]

\[ |\vec{u}| = \sqrt{\vec{u} \cdot \vec{u}} \]
Find the angle between a diagonal of a cube and an edge.

\[ \vec{u} = \hat{i} + \hat{j} + \hat{k} \]
\[ \vec{v} = \hat{k} \]

\[ \cos(\theta) = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \|\vec{v}\|} = \frac{1}{\sqrt{3}} \]
The Cube

- Emphasizes that vectors are arrows
- Combines geometry and algebra
- Uses multiple representations

\[
\vec{u} \cdot \vec{v} = \|\vec{u}\| \|\vec{v}\| \cos \theta
\]

geometry:

\[
\vec{u} \cdot \vec{v} = (u_1 \hat{i} + u_2 \hat{j} + u_3 \hat{k}) \cdot (v_1 \hat{i} + v_2 \hat{j} + v_3 \hat{k})
\]

algebra:

\[
\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 + u_3 v_3
\]

memory: