The problem of coexistence in multi-type competition models

Abstract. We consider a two-type stochastic competition model on the integer lattice $\mathbb{Z}^d$. The model describes the space evolution of two “species” competing for territory along their boundaries. Each site of the space may contain only one representative (also referred to as a particle) of either type. The spread mechanism for both species is the same: each particle produces offspring independently of other particles and can place them only at the neighboring sites that are either unoccupied, or occupied by particles of the opposite type. In the second case, the old particle is killed by the newborn. The rate of birth for each particle is $\lambda \times (\# \text{ of neighboring sites available for expansion})$, for $\tau = 1, 2$ corresponding to the types 1 and 2. The main problem we address concerns the possibility of the long-term coexistence of the two species. We have shown that if we start the process with finitely many representatives of each type, then, assuming $\lambda_1 = 1$, there exists a critical rate $\lambda_c > 1$ such that, for all $\lambda_2 \in (1, \lambda_c)$, there is a positive probability of coexistence. For all $\lambda_2 \in (\lambda_c, \infty)$ only one type of particle can survive.

For the case where $\lambda_1 = \lambda_2$ we were able to prove the coexistence under certain conditions on the shape of the limit set in the first passage percolation model. We also conjecture that the regions colonized by the two types stabilize as $t \to \infty$.

Related models on the integer lattice and homogeneous trees are described along with some open problems and conjectures.

This is joint work with Steven Lalley.