1. (14 points) Suppose that an object of mass 10 kg with drag coefficient $\gamma = 2 \text{ kg/sec}$ is dropped from a height of 300 m. Assuming that the acceleration due to gravity is $9.8 \text{ m/sec}^2$, the terminal velocity (in m/sec) is

A. $v = 9.8$
B. $v = 245$
C. $v = 50$
D. $v = 49$
E. $v \approx 43.01$

2. (12 points) The first order linear equation $t^3y' + y = \tan^2(t)$ admits the integrating factor

A. $\mu(t) = \exp\left(\frac{1}{2t^2}\right)$
B. $\mu(t) = \exp\left(-\frac{1}{t^3}\right)$
C. $\mu(t) = \exp\left(-\frac{1}{2t^2}\right)$
D. $\mu(t) = \exp(t^3)$
E. $\mu(t) = \exp(-t)$
3. (12 points) The general solution of the separable equation \( y' = \frac{x}{y(1 + x^2)} \) for \( y \neq 0 \) is

A. \( y = \frac{2x}{1 + x^2} + C \)
B. \( y = \ln(1 + x^2) + C \)
C. \( y^2 = \arctan(x) + C \)
D. \( y^2 = \ln(1 + x^2) + C \)
E. \( y = \arctan(x) + C \)

4. (12 points) The autonomous ODE \( y' = y(1 - y^2) \) possesses asymptotically stable equilibrium solution(s)

A. \( y^* = 1 \)
B. \( y^* = 0 \)
C. \( y^* = -1, \ y^* = 1 \)
D. \( y^* = -1, \ y^* = 0, \ y^* = 1 \)
E. \( y^* = -1 \)
5. (12 points) The differential equation \( y^2 + (2xy + 1)y' = 0 \) is exact. If we write it in the form

\[
\frac{\partial \psi}{\partial x} + \frac{\partial \psi}{\partial y} \frac{dy}{dx} = 0,
\]

then \( \frac{d\psi}{dx} = 0 \) leads to the implicitly defined solution to the ODE

A. \( \psi(x, y) = y^2x + 1 = C \)
B. \( \psi(x, y) = y^2 = C \)
C. \( \psi(x, y) = y^2x = C \)
D. \( \psi(x, y) = 2xy = C \)
E. \( \psi(x, y) = y^2x + y = C \)

6. (12 points) The nonlinear differential equation \( \frac{y^2}{x} + \left(2y + \frac{1}{x}\right)y' = 0 \) is not exact, however, it may be converted into an exact ODE through multiplying by the integrating factor

\[
\mu(x) = \exp \left( \int Q(x)dx \right)
\]

where the function \( Q(x) \) is

A. \( 1/x \)
B. \( 1 \)
C. \( x \)
D. \( \ln(x) \)
E. \( -x \)
7. (12 points) A small population of deer reproduces each year with growth rate $\rho = 1.5$ (one fawn per couple), and the environment imposes a (discrete) carrying capacity $k = 300$. Hunters further reduce the population by $b = 25$ per year. The discrete logistic model for this population, given by

$$y_{n+1} = \rho y_n \left(1 - \frac{y_n}{k}\right) - b,$$

possesses equilibrium solution(s):

A. $y^* = 0, \ y^* = 600$
B. $y^* = 100$
C. $y^* = -100, \ y^* = 100$
D. $y^* \approx 17, \ y^* \approx 583$
E. $y^* = 600$

8. (14 points) The initial value problem $y'' + 3y' + 2y = 0, \ y(0) = 2, \ y'(0) = -1$ has the solution

A. $y = 3e^{-2t} - e^{-t}$
B. $y = -e^{-2t} + 3e^{-t}$
C. $y = -3e^{2t} + 5e^t$
D. $y = 5e^{2t} - 3e^t$
E. $y = -e^{-2t} + 5e^t$
9. (12 points) The initial value problem \( t^3y' + y = \tan^2(t) \), \( y(1) = 2 \)

A. has a solution on the interval \(-\pi/2 < t < \pi/2\), but the solution is not unique
B. has a solution on the interval \(-\pi/2 < t < 0\), but the solution is not unique
C. has a unique solution, but this solution only exists on the interval \(0 < t < \pi/2\)
D. has a solution on the interval \(0 < t < \pi/2\), but the solution is not unique
E. has a unique solution, but this solution only exists on the interval \(-\pi/2 < t < \pi/2\)

10. (12 points) Let \( y_1(t) = \sin t \), \( y_2(t) = \cos t \), \( y_3(t) = t \). Then the Wronskian of \( y_1(t), y_2(t), y_3(t) \) is

A. \( W(y_1, y_2, y_3)(t) = -t \)
B. \( W(y_1, y_2, y_3)(t) = \sin^2 t - \cos^2 t \)
C. \( W(y_1, y_2, y_3)(t) = t(\sin^2 t - \cos^2 t) \)
D. \( W(y_1, y_2, y_3)(t) = \sin t \cos t \)
E. \( W(y_1, y_2, y_3)(t) = t \sin t \cos t \)
11. (14 points) The characteristic polynomial of a constant coefficient linear, homogeneous differential equation is \((r^2 + 1)^2(r + 2)\). Then the general solution to the equation is

A. \(c_1 \sin t + c_2 \cos t + c_3 t \sin t + c_4 t \cos t + c_5 e^{-2t}\)
B. \(c_1 t \sin t + c_2 t \cos t + c_3 e^{-2t}\)
C. \(c_1 \sin t + c_2 \cos t + c_3 e^{-2t}\)
D. \(c_1 e^t + c_2 e^{-t} + c_3 t e^t + c_4 t e^{-t} + c_5 e^{-2t}\)
E. \(c_1 t e^t + c_2 t e^{-t} + c_3 e^{-2t}\)

12. (12 points) Suppose you are using the method of undetermined coefficients to find a particular solution to the equation \(y'' + 2y' + y = e^{-t} + t\). The suitable form \(Y(t)\) for the particular solution is

A. \(Y(t) = At + Bt^2 + Ct^2 e^{-t}\)
B. \(Y(t) = At + Ce^{-t}\)
C. \(Y(t) = A + Bt + Ce^{-t}\)
D. \(Y(t) = A + Bt + Ct^2 e^{-t}\)
E. \(Y(t) = A + Bt + Cte^{-t}\)
13. (14 points) The differential equation \( xy'' - (1 + x)y' + y = 0 \) possesses the solutions \( y_1(x) = 1 + x, \ y_2(x) = e^x \). Suppose you are finding a particular solution to \( xy'' - (1 + x)y' + y = \sin x \) in the form \( y_p = u_1(x)(1 + x) + u_2(x)e^x \) using the method of variation of parameters. Then the functions \( u_1(x), \ u_2(x) \) satisfy the equations

A. \( u_1'(x) + e^xu_2'(x) = \sin x, \quad u_1'(x) + e^xu_2'(x) = 0 \)

B. \( u_1'(x) + e^xu_2'(x) = \sin x, \quad (1 + x)u_1'(x) + e^xu_2'(x) = 0 \)

C. \( xu_1'(x) + xe^xu_2'(x) = \sin x, \quad (1 + x)u_1'(x) + e^xu_2'(x) = 0 \)

D. \( xu_1'(x) + xe^xu_2'(x) = 0, \quad (1 + x)u_1'(x) + e^xu_2'(x) = \sin x \)

E. \( xu_1'(x) + xe^xu_2'(x) = \sin x, \quad u_1'(x) + e^xu_2'(x) = 0 \)

14. (12 points) Let \( y(t) \) be the solution to the initial value problem \( y'' + 2y' + 3y = \cos t, \ y(0) = 2, \ y'(0) = -1 \). Then the Laplace transform of \( y(t) \) is

A. \( Y(s) = \frac{2s - 3}{s^2 + 2s + 3} + \frac{s}{(s^2 + 1)(s^2 + 2s + 3)} \)

B. \( Y(s) = \frac{s}{(s^2 + 1)(s^2 + 2s + 3)} \)

C. \( Y(s) = \frac{3 + 2s}{s^2 + 2s + 3} + \frac{s}{(s^2 + 1)(s^2 + 2s + 3)} \)

D. \( Y(s) = \frac{6 - s}{s^2 + 2s + 3} + \frac{s}{(s^2 + 1)(s^2 + 2s + 3)} \)

E. \( Y(s) = \frac{3 + 2s}{s^2 + 2s + 3} \)
15. (6 points) The Laplace transform of \( f(t) = \cosh 2t = \frac{e^{2t} + e^{-2t}}{2} \) is

A. \( F(s) = \frac{4}{s^2 + 4} \)

B. \( F(s) = \frac{4}{s^2 - 4} \)

C. \( F(s) = \frac{s}{s^2 - 4} \)

D. \( F(s) = \frac{4}{s^2 + 4} \)

E. \( F(s) = \frac{s}{s^2 - 2} \)

16. (6 points) The inverse Laplace transform of \( F(s) = e^{-\pi s/4} \frac{8}{s^2 + 1} \) is

A. \( f(t) = \sin(t - \pi/4) \)

B. \( f(t) = u_{\pi/4}(t) \cos(t - \pi/4) \)

C. \( f(t) = u_{\pi/4}(t) \sin(t - \pi/4) \)

D. \( f(t) = \cos(t - \pi/4) \)

E. \( f(t) = \tan(t - \pi/4) \)

17. (12 points) The function \( y(t) = 1/t \) is a solution of \( 2t^2 y'' + 3ty' - y = 0 \). Suppose you are using reduction of order to find another solution to the equation in the form \( y_2(t) = v(t)y_1(t) \). Then \( v(t) \) satisfies the differential equation

A. \( 2tv'' - v' = 0 \)

B. \( v'' + \frac{v'}{t} = 0 \)

C. \( 2tv' - v = 0 \)

D. \( 2tv'' - v' - v = 0 \)

E. \( v' + \frac{v}{t} = 0 \)