1. A population of fish being harvested is modeled by $\frac{dp}{dt} = 2p - 100$, where $p > 0$ is the population. Suppose that initially the population $p_0 \neq 50$. As time increases, the population
   A. converges to $p = 50$
   B. diverges from $p = 50$
   C. goes to zero
   D. diverges from $p = 50$ if $p_0 < 50$, otherwise converges
   E. diverges from $p = 50$ if $p_0 > 50$, otherwise converges

2. The first order linear equation $y' - \sin(t) y = 4 - t$ admits the integrating factor
   A. $\mu(t) = e^{-\sin(t)}$
   B. $\mu(t) = e^{\sin(t)}$
   C. $\mu(t) = e^{-\cos(t)}$
   D. $\mu(t) = e^{\cos(t)}$
   E. $\mu(t) = e^{-t \sin(t)}$

3. The equation
   $$\frac{d^2y}{dx^2} = y^3$$
   can best be described as a
   A. second order, nonlinear ODE
   B. third order, linear ODE
   C. first order, linear ODE
   D. third order, nonlinear PDE
   E. second order, linear PDE
4. Given the differential equation

\[ ty' - y = t^2 e^{-t}, \quad t > 0 \]

the general solution is

A. \[ y = -(2 + 2t + t^2) e^{-t} + ct \]
B. \[ y = -te^{-t} + ct \]
C. \[ y = cte^{-t} \]
D. \[ y = -e^{-t} + c \]
E. \[ y = -ce^{-t} \]

5. Given the initial value problem

\[ y' = \frac{1}{2} y^2 - xy^2, \quad y(0) = 2 \]

the solution is

A. \[ y = \frac{1}{\frac{1}{2}x^2 - x + \frac{1}{2}} \]
B. \[ y = \frac{2}{x^2 + \frac{1}{2}x + 1} \]
C. \[ y = \frac{2}{x^2 - x + 1} \]
D. \[ y = \frac{1}{x^2 - x + \frac{1}{2}} \]
E. \[ y = \frac{1}{\frac{1}{2} + x - x^2} \]

6. The general solution of the separable equation \( y' = t^2/y \) is

A. \[ y = \frac{t^3}{3y} + C \]
B. \[ y = t^2 + C \]
C. \[ y = \frac{2}{3}t^3 + C \]
D. \[ y^2 = t^2 + C \]
E. \[ y^2 = \frac{2}{3}t^3 + C \]
7. The initial value problem $y' + t^3 y = \tan^2 t$, $y(0) = 2$

A. has a unique solution, but this solution only exists on the interval $-\pi/2 < t < \pi/2$
B. has a solution on the interval $-\pi/2 < t < \pi/2$, but the solution is not unique
C. has a unique solution which is defined everywhere
D. does not have a solution
E. has a solution which is defined everywhere, but the solution is not unique

8. A toxic chemical has been spilled into a lake with volume $V$ gallons. Fresh water flows into the lake at rate $r$ gallons/day, the flow out of the lake is at the same rate. Assuming the chemical is evenly distributed throughout the lake, a model for the amount of chemical $Q$ in the lake is given by

A. $\frac{dQ}{dt} = -rQ$
B. $\frac{dQ}{dt} = -rQV$
C. $\frac{dQ}{dt} = -\frac{rQ}{V}$
D. $\frac{dQ}{dt} = \frac{r}{V} - \frac{rQ}{V}$
E. $\frac{dQ}{dt} = rV - rQ$

9. The differential equation $(3xy + y^2) + (x^2 + xy)y' = 0$, which is not exact, admits the integrating factor

A. $\mu(x, y) = xy$
B. $\mu(x) = 1/x$
C. $\mu(y) = y$
D. $\mu(x) = x$
E. $\mu(y) = 1/y$

10. Initially the population of a certain species of fish in a lake is 1,000. The intrinsic growth rate (in years) of the population is estimated to be 0.1 and the environmental carrying capacity to be 3,000 fish. By the logistic model, two years later the fish population in the lake will be approximately [Hint: $y = y_0K/(y_0 + (K - y_0)e^{-rt})$]

A. 702
B. 1,649
C. 871
D. 1,137
E. 1,832
11. The general solution to the differential equation \(y'' - y' = 0\) is
   A. \(y(t) = C_1 + C_2 e^t\)
   B. \(y(t) = C_1 e^t + C_2 e^{-t}\)
   C. \(y(t) = C_1 + C_2 e^{-t}\)
   D. \(y(t) = C_1 t + C_2 e^t\)
   E. \(y(t) = C_1 + C_2 t\)

12. The functions \(y_1(t) = t^2, \ y_2(t) = 1/t\) satisfy the differential equation \(t^2 y'' - 2y = 0\). The solution to the initial value problem
\[
t^2 y'' - 2y = 0, \quad y(1) = 1, \quad y'(1) = 0
\]
is
   A. \(y(t) = \frac{t^2}{3} + \frac{2}{3t}\)
   B. \(y(t) = \frac{4t^2}{3} - \frac{2}{3t}\)
   C. the differential equation does not have a solution with the specified initial conditions.
   D. \(y(t) = t^2\)
   E. \(y(t) = \frac{t}{3} + \frac{1}{3t}\)

13. Let \(y_1(t) = t, \ y_2(t) = te^t\). Then the Wronskian of \(y_1(t)\) and \(y_2(t)\) is
   A. \(W(y_1, y_2)(t) = t\)
   B. \(W(y_1, y_2)(t) = 2t\)
   C. \(W(y_1, y_2)(t) = te^t\)
   D. \(W(y_1, y_2)(t) = t^2 e^t\)
   E. \(W(y_1, y_2)(t) = 0\)