1. Approximate each of the following integrals using the trapezoidal rule:

i. \[ \int_0^2 e^{-x^2} \, dx \]

ii. \[ \int_0^4 \frac{1}{1 + x^2} \, dx \]

iii. \[ \int_0^1 \sqrt{x} \, dx \]

(a) For each integral, create a table of values \( T_n(f) \) for \( n = 2, 4, 8, \ldots, 512 \). Also compute the difference between successive iterates \( T_n(f) - T_{n-1}(f) \), and the ratio between successive differences \( (T_n(f) - T_{n-1}(f))/(T_{n-1}(f) - T_{n-2}(f)) \).

Your table should look something like:

<table>
<thead>
<tr>
<th>n</th>
<th>Approximation</th>
<th>Difference</th>
<th>Ratio</th>
</tr>
</thead>
</table>

Note: In order to see all significant figures, it is helpful to use something similar to the following when you output values in Matlab:

```matlab
[inT,diT,raT]=trapezoidal(a,b,n0,indexf);
for i=1:length(inT),
    disp(sprintf('%d 	%0.12f 	%0.5e 	%g', n0*2^(i-1),inT(i),diT(i),raT(i)))
end
```

(b) Comment on whether the trapezoidal rule performed as well as expected for each integral. If it did not, explain what may be the cause.

2. Repeat 1 using Simpson’s rule.

3. Regarding integral (i.), the asymptotic error formula for Simpson’s rule estimates that the number of subdivisions required to achieve an accuracy of \( \epsilon = 10^{-10} \) is at least \( n = 160 \). For integral (ii.) \( n = 396 \) is required for an accuracy of \( \epsilon = 10^{-12} \). Comment on whether your computational results agree or disagree with the asymptotic error formula.

Note: Turn in a hard copy of the Matlab script(s), Matlab output including tables, along with your answers to the questions for each problem.