1. Consider the $3 \times 3$ matrix

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Compute $||A||_1, ||A||_\infty, ||A||_F$.

2. For the matrix $A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$

(a) Compute the SVD of $A$.

(b) Plot the right and left singular vectors.

(c) Show that the singular values of $A$ are the absolute values of the eigenvalues of $A$.

(d) What is $||A||_2$?

(e) What is the 2-norm condition number of $A$?

(f) What is the rank of $A$?

3. Do any two of the following:

(a) Let $|| \cdot ||$ be any norm on $\mathbb{C}^n$ as well as the induced matrix norm on $\mathbb{C}^{n \times n}$. Let $A$ be an $n \times n$ matrix. Show that

$$\rho(A) \leq ||A||$$

where $\rho(A) = \max\{||\lambda||; \lambda \text{ an eigenvalue of } A\}$.

(b) Let $u \in \mathbb{C}^m$ and $v \in \mathbb{C}^n$. Prove that $||uv^*||_2 = ||u||_2 ||v||_2$.

(c) Let $A$ be an $n \times n$ matrix. Define $||A||_{\text{max}} = \max_{1 \leq i,j \leq n}|a_{ij}|$. Show that $||A||_{\text{max}}$ is a matrix norm.