1. Read examples 1.2 and 3.6 in Trefethen and Bau. Prove that $\forall u \in \mathbb{C}^m$ and $\forall v \in \mathbb{C}^n$
$$
\|uv^*\|_F = \|u\|_2\|v\|_2
$$
Also, show by an example that
$$
\|uv^*\|_F \neq \|u\|_F\|v\|_F
$$

2. Aside from the $p$-norms, the most useful norms are the *weighted $p$-norms*, where each of the coordinates of a vector space is given its own weight. Given any norm, prove that the function $\| \cdot \|_W$, where $W$ is an arbitrary nonsingular matrix, given by
$$
\|x\|_W = \|Wx\|
$$
is a vector norm.

3. Prove that the $\infty$-norm of an $m \times n$ matrix is equal to the maximum row sum, i.e.,
$$
\|A\|_\infty = \max_{1 \leq i \leq m}\|a_i^*\|_1,
$$
where $a_i^*$ denotes the $i$th row of $A$.

4. Do [TB] 4.1

5. Do [TB] 5.3

6. Do [TB] 7.1

7. Let $A$ be an $m \times n$ matrix. Determine the exact numbers of floating point additions, subtractions, multiplications, and divisions involved in computing the factorization $A = Q\hat{R}$ by Algorithm 7.1.