1. (452) 5.2 from [AP]. For part (a), use Example 5.6. For each of the values of \( \lambda \) given, plot the exact solution and all four numerical solutions on one graph, with a legend to distinguish between them.

2. (452) Solve

\[ y'(t) = 3y(t), \quad y(0) = 1 \]

with the Leapfrog method (two-step midpoint method)

\[ y_n = y_{n-2} + 2hf_{n-1} \]

in two different ways.

(a) Generate \( y_1 \) using the Forward Euler method. Then calculate \( y_2, y_3 \ldots \) etc. using the Leapfrog method. Calculate the resulting errors at time \( T = 1 \) for \( h = 2^{-j}, j = 5, 6, 7, 8, 9, 10 \). Also calculate the ratio of errors for successive values of \( h \).

(b) Generate \( y_1 \) using the Trapezoidal method. Then calculate \( y_2, y_3 \ldots \) etc. using the Leapfrog method. Calculate the resulting errors at time \( T = 1 \) for \( h = 2^{-j}, j = 5, 6, 7, 8, 9, 10 \). Also calculate the ratio of errors for successive values of \( h \).

Assemble your results in a table which includes the value of \( h \), the errors using the Forward Euler/Leapfrog and Trapezoidal/Leapfrog methods, as well as ratios of successive errors in each case. What are the differences observed? What can you conclude from this experiment?

3. Show directly by expanding the terms in \( y(t_{n-j}) \) and \( f(t_{n-j}, y(t_{n-j})) \), how to construct the coefficient \( C_2 \) and \( C_3 \) used in the verification of the consistency order of a multistep method. (In class we constructed the coefficients \( C_0 \) and \( C_1 \).)

4. (a) (452) Adams methods can be easily generalized by integrating between \( t_{n-k} \) and \( t_n \), with \( k \geq 2 \). Show that, by doing so, we get methods of the form

\[ y_n = y_{n-k} + h \sum_{j=0}^{p} \beta_j f_{n-j} \]

(b) Let \( k = 2 \). The explicit methods in this class are called Nyström methods and the implicit methods are called Generalized Milne-Simpson methods. Consider the Leapfrog method (two-step midpoint method)

\[ y_n = y_{n-2} + 2hf_{n-1}, \]
which is an example of a Nyström method. How is this method constructed? (discuss interpolating polynomial etc.) What are the characteristic polynomials for this method? Discuss consistency order, 0-stability and (region of) absolute stability of this method.

(c) Consider the Milne-Simpson method

\[ y_n = y_{n-2} + \frac{h}{3}(f_n + 4f_{n-1} + f_{n-2}) \]

What are the characteristic polynomials for this method? Discuss consistency order, 0-stability and (region of) absolute stability of this method.

5. Consider the BDF method

\[ y_n = \frac{4}{3}y_{n-1} - \frac{1}{3}y_{n-2} + \frac{2b}{3}f_n \]

Discuss consistency order, 0-stability and (region of) absolute stability of this method.