Homework assignment 2*

Due date: October 30, 2006.

1. The period of Jupiter's orbit is 11.86 earth years. What is the length of its semi-major axis? Compare it to the length of Earth's semi-major axis. Keep this ratio in mind next time when you drive on NW 8 th Ave (in Eastern direction, starting from the intersection with 34 th Street) and see the scaled version of the solar system. Does it look like the designers of the scaled version actually scaled accurately?

2. Find the general solution of
$$y'' - y' - 6y = t^2\,\mathrm{e}^{3t}\,.$$

3. Consider an RLC-circuit (see Fig 4.9 on p. 178 of our text) with $R = 3$ (Ohms), $L = 1$ (Henrys) and $C = 1/2$ (Farads), and $E(t) = \cos(t)$ (Volts). Find an ODE for q, the charge on one of the plates of the capacitor. Assume that at $t = 0$, the initial charge on this plate is 1 (Coulomb) and that the initial current is 0 (Ampères). Determine $q(t)$.

4. Consider an externally forced mass-spring system (no damper). It obeys the equation:
$$m\frac{d^2x}{dt^2} + kx = A\cos(\omega_1 t),$$
where m is the mass, k is the spring's constant (cfr Hooke's law). The external force is periodic with amplitude $A > 0$ and frequency $\omega_1/2\pi$. If there is no external forcing ($A = 0$), we get a harmonic oscillator with natural frequency $\omega_0/2\pi$, where $\omega_0 = \sqrt{k/m}$.

   Show that the general solution $y_g(t)$ is bounded if $\omega_1 \neq \omega_0$ (a function $f(t)$ is called bounded if there is some positive constant K so that $|f(t)| < K$ for all t.), but that $y_g(t)$ is not bounded if $\omega_1 = \omega_0$. This last case is a typical example of what is known as a *resonance phenomenon*. Such phenomena may occur when a frequency of an external forcing term matches a natural frequency of the unforced system.

5. Find the general solution to the system
$$\begin{aligned} \frac{dx}{dt} &= x + y \\ \frac{dy}{dt} &= 2x + y \end{aligned}$$

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