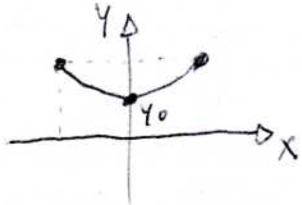
3. 

$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{w}{T}\sqrt{1+\left(\frac{dy}{dx}\right)^2}$. Put $v = \frac{dy}{dx}$, so $\frac{dv}{dx} = \frac{w}{T}\sqrt{1+v^2}$, separable

$\Rightarrow \frac{dv}{\sqrt{1+v^2}} = \frac{w}{T}dx \overset{(*)}{\Rightarrow} \sinh^{-1} v = \frac{w}{T}x + C$ or $v = \sinh\left(\frac{w}{T}x + C\right) = \sinh\left(\frac{w}{T}x\right)$

$v(0) = \frac{dy}{dx}(0) = 0 \Rightarrow 0 = \sinh C \Rightarrow \boxed{C=0}$

(*) pf: Will show $\int \frac{dv}{1+v^2} = \sinh^{-1} v$

$\sinh(\sinh^{-1} v) = v$

$\Rightarrow$ implicit differentiation: $(\sinh^{-1} v)' \cdot \cosh(\sinh^{-1} v) = 1$

Now $\cosh^2 v - \sinh^2 v = 1$, so $\cosh v = +\sqrt{1+\sinh^2 v}$

$\cosh v$ never negative

$\Rightarrow (\sinh^{-1} v)' = \frac{1}{\sqrt{1+v^2}}$, or

equivalently: $\int \frac{dv}{\sqrt{1+v^2}} = \sinh^{-1} v$

So $\frac{dy}{dx} = \sinh\left(\frac{w}{T}x\right) \Rightarrow \boxed{y(x) = \frac{T}{w}\cosh\left(\frac{w}{T}x\right) + C' = \frac{T}{w}\left(\cosh\left(\frac{w}{T}x\right) - 1\right) + y_0}$

$y(0) = y_0 \Rightarrow C' = y_0 - \frac{T}{w}$

4. Only for $y = cx^2$ (parabolas)

$\frac{dy}{dx} = 2cx \Rightarrow$ orthogonal family: $\frac{dy}{dx} = -\frac{1}{2cx} \overset{c = \frac{y}{x^2}}{=} -\frac{1}{2\cdot\frac{y}{x^2}\cdot x} = -\frac{x}{2y} \overset{\text{separable}}{\Rightarrow} 2y\,dy = -x\,dx$

$\Rightarrow y^2 = -\frac{x^2}{2} + k$

$y^2 + \frac{x^2}{2} = k$, ellipses $(k>0)$

$$\boxed{\left(\frac{y}{\sqrt{k}}\right)^2 + \left(\frac{x}{\sqrt{2}\sqrt{k}}\right)^2 = 1}$$