5. $(*)\begin{cases} \dfrac{dx}{dt} = ax + by \\ \dfrac{dy}{dt} = cx + dy \end{cases}$

- $z = \dfrac{y}{x} \Rightarrow \dfrac{dz}{dt} = \dfrac{\frac{dy}{dt}\cdot x - y\cdot\frac{dx}{dt}}{x^2}$

$$\overset{(*)}{=} \frac{(cx+dy)x - y(ax+by)}{x^2}$$

$$= c + d\frac{y}{x} - a\frac{y}{x} - b\left(\frac{y}{x}\right)^2$$

$$= c + (d-a)z - bz^2$$

- So $\dfrac{dz}{dt} = c + (d-a)z - bz^2$, nonlinear, order 1 (= highest order of derivatives appearing in the differential equation)

- $z_p(t)$: particular solution, i.e. $\dfrac{dz_p}{dt} = c + (d-a)z_p - bz_p^2$

$$z = z_p + \frac{1}{q} \Rightarrow \frac{dz}{dt} = \frac{dz_p}{dt} - \frac{1}{q^2}\frac{dq}{dt}$$

or $\cancel{c} + (d-a)z - bz^2 = \cancel{c} + (d-a)z_p - bz_p^2 - \dfrac{1}{q^2}\dfrac{dq}{dt}$

$$\Rightarrow (d-a)\underbrace{(z - z_p)}_{=\frac{1}{q}} - b\underbrace{(z - z_p)}_{=\frac{1}{q}}\underbrace{(z + z_p)}_{=2z_p + \frac{1}{q}} = -\frac{1}{q^2}\frac{dq}{dt}$$

$$\Rightarrow (d-a)\cdot\frac{1}{q} - (2bz_p(t))\cdot\frac{1}{q} - b\cdot\frac{1}{q^2} = -\frac{1}{q^2}\frac{dq}{dt}$$

$\cdot q^2 \Rightarrow$ $\boxed{\dfrac{dq}{dt} = b + (2bz_p(t) - (d-a))q}$, linear equation