Homework assignment 1*

Due date: September 18

1. **Legendre's equation**

   Find the first 7 terms of the power series expansion about $x = 0$ of the general solution to Legendre's equation:

   $$(1 - x^2)y'' - 2xy' + p(p + 1)y = 0,$$

   where p is an arbitrary real parameter.

   These general solutions are called *Legendre's functions*. Explain why they are called *Legendre polynomials* in case p is a positive integer.

2. **Method of Frobenius I**

   Determine the form of a series expansion about $x = 0$ of 2 linearly independent solutions to

   $$xy'' - sy' + x^3y = 0,$$

   where s is an arbitrary real number. Your answer should depend on the value of s.

3. **Method of Frobenius II**

   Find the first 3 terms of the series expansion about $x = 0$ of 2 linearly independent solutions to

   $$x^2y'' - x^2y' + (x^2 - 2)y = 0$$

4. **Property of the Gaussian hypergeometric function.**

   Denoting the Gaussian hypergeometric function by $F(\alpha, \beta, \gamma; x)$, show that

   $$\ln(1 + x) = xF(1, 1, 2; -x).$$

5. **Properties of Bessel functions.**

   Denoting the Bessel function of the first kind of order $\nu > 0$ by $J_\nu(x)$, show that the following properties hold:

   $$\frac{d}{dx}\left(x^{-\nu}J_\nu(x)\right) = -x^{-\nu}J_{\nu+1}(x) \text{ and } J_{\nu+1}(x) = \frac{2\nu}{x}J_\nu(x) - J_{\nu-1}(x).$$

*MAP 4305; Instructor: Patrick De Leenheer.