Homework assignment 2*

Due date: October 30. 2006.

1. Determine all real eigenvalues and corresponding eigenfunctions of

$$y'' + \lambda y = 0, \quad y(0) = 0, \quad y(\pi) + y'(\pi) = 0.$$

2. Determine the adjoint problem of

$$y'' + y' - 2y = 0, \quad y(0) + y'(0) = 0, \quad y(1) + y'(1) = 0.$$

   Is the given problem self-adjoint?

3. Consider the following boundary value problem:

$$y'' + y = \sin 2x, \quad y(0) = y(2\pi), \quad y'(0) = y'(2\pi).$$

   - Using the Fredholm alternative, determine whether or not this problem has solutions.
   - If there are solutions, determine them.

4. Is

$$\begin{pmatrix} e^{-3t} & e^{-3t} + 3\,e^{3t} & e^{3t} \\ e^{-3t} & e^{-3t} - 3\,e^{3t} & -e^{3t} \\ -e^{-3t} & -e^{-3t} & 0 \end{pmatrix}$$

   a fundamental matrix solution of

$$\dot{x} = \begin{pmatrix} 1 & -2 & 2 \\ -2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix} x ?$$

5. Find a fundamental matrix solution of

$$\dot{x} = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix} x$$