Practice Exam 2: MAP 4305*

1. Find the **positive** eigenvalues and eigenfunctions of the following eigenvalue problem:
$$y'' + \lambda y = 0, \;\; y(0) + y'(0) = y(\pi) = 0.$$

2. Find conditions on f so that the following non-homogeneous BV problem has a solution:
$$y'' - y' + 3y = f, \;\; y(0) = y(\pi) = 0$$

3. Are the vector functions
$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \begin{pmatrix} t \\ t \\ t \end{pmatrix}, \quad \begin{pmatrix} t^2 \\ t^2 \\ t^2 \end{pmatrix},$$
defined for $t \in \mathbb{R}$, linearly independent? If yes, can they be a fundamental solution set of a system $\dot{x} = A(t)x$ with $x \in \mathbb{R}^3$?

4. Solve the following IVP:
$$\dot{x} = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} x, \;\; x(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

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