Homework assignment 3*

Due date: Friday November 9, 2007

1. Consider the following model of a bioreactor with two species with concentrations x and y:

$$\begin{aligned} \dot{x} &= x(f(1-x-y)-1) \\ \dot{y} &= y(g(1-x-y)-1), \end{aligned}$$

   where $(x,y) \in T = \{(x,y) \in \mathbb{R}^2 \,|\, x > 0,\, y > 0,\; x+y < 1\}$. The functions f and g are differentiable functions with $f(0) = g(0) = 0$ with f', $g' > 0$. Show that this system has no non-trivial periodic solutions in T.

2. In class we discussed the van der Pol oscillator which we showed could be written as follows:

$$\begin{aligned} \dot{x} &= x+y-x^3/3 \\ \dot{y} &= -x \end{aligned}$$

   We showed that the van der Pol oscillator has a nontrivial periodic solution in a certain annular region that did not contain the origin. We did not completely *prove* that this region is a trapping region, and the purpose of this problem is to fill the gap by doing the following:

   Prove that the polygonal region R depicted below is a trapping region.

3. Find the first three terms of two linearly independent solutions of

$$2x(x-1)y'' + 3(x-1)y' - y = 0, \quad x > 0.$$

4. Consider Laguerre's equation:

$$xy'' + (1-x)y' + ny = 0,$$

   where n is a nonnegative integer.

   Show that for every n, this equation has a polynomial solution $L_n(x)$ and determine L_0, L_1, L_2 and L_3 (This is problem 24 from section 8.7 in our text).

5. Do problem 22 of section 8.7 in our text.

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