Practice exam I: MAP 4305*

September 11, 2006.

Name:
Student ID:

This is a **closed book** exam and the use of calculators is **not** allowed.

1. Let
$$A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}.$$
   (a) Calculate A^{231}.
   (b) Discuss stability for the system $x(t+1) = Ax(t)$.

2. Consider a network with two nodes (call them 1 and 2) and assume that there is a link from 1 to 2 (but not back), and that both 1 and 2 have a self-link. Calculate the pagerank for both nodes. (To determine the stochastic matrix, use the same rule we used on the HW problem).

3. Suppose that the vector functions
$$\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} \text{ and } \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix},$$
   where $t \in \mathbb{R}$, are solutions of
$$\dot{x} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} x.$$
   Let $X(t)$ be defined as:
$$\begin{pmatrix} x_1(t) & y_1(t) \\ x_2(t) & y_2(t) \end{pmatrix}$$
   and define the quantity $m(t)$ as the determinant of the matrix $X(t)$. Show that $m(t)$ satisfies the following differential equation:
$$\dot{m}(t) = (a+d)m(t).$$
   Show that this implies that
$$m(t) = \mathrm{e}^{(a+d)t}\, m(0).$$
   Hint: Write $m(t)$ explicitely in terms of $x_1(t), x_2(t), y_1(t)$ and $y_2(t)$, and calculate the derivative with respect to time t.

4. Solve the following IVP:
$$\dot{x} = \begin{pmatrix} -1 & 1 \\ 0 & 2 \end{pmatrix} x, \quad x(1) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

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