Practice Exam 3: MAP 4305*

1. Does
$$5xy'' + 4(1-x^2)y' + y = 0, \quad x > 0,$$
have a solution which is bounded near zero? Notice that to answer this question, you only need to consider the indicial equation.

2. Determine **the form** of a series expansion about $x = 0$ of 2 linearly independent solutions to:
$$x^2y'' - xy' + (1-x^2)y = 0, \quad x > 0.$$
Do not find a recursion formula for the coefficients.

3. Find the first three non-zero terms in a series expansion about $x = 0$ of 2 linearly independent solutions to:
$$3xy'' + (2-x)y' - y = 0, \quad x > 0.$$

4. Draw solutions in the (x, y) plane of the following system in polar coordinates:
$$\begin{aligned} \dot{r} &= \sin r \\ \dot{\theta} &= -1 \end{aligned}$$
Are there any non-trivial periodic solutions? If yes, are they limit cycles? If there are non-trivial periodic solutions, how many are there, and what can be said about their stability?

5. The Legendre polynomials $P_n(x)$ satisfy the following recurrence relation:
$$(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x).$$
Given that $P_0(x) = 1$ and $P_1(x) = x$, determine $P_2(x)$, $P_3(x)$ and $P_4(x)$.

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