Name:

This is a **closed book** exam and the use of formula sheets or calculators is **not** permitted.

The first 6 problems are multiple choice problems, **each worth 5 points**. Please circle the right answer (there is exactly one for each problem). The last 2 problems require that you show all work, and will receive partial credit. They are worth **15 points each**.

1. Let

   \[ f(x) = \frac{1}{2} + x^3, \quad 0 \leq x \leq 4. \]

   We wish to approximate the area under the graph of \( f \) and above the horizontal axis, when \( x \) varies between 0 and 4. To do so, we partition \([0, 4]\) in 4 subintervals of length 1, and use the function value in the left endpoint of each subinterval. The approximate area is:

   - 38. A
   - 66.
   - 102.
   - 129/2.

2. A car’s velocity (measured in miles per hour) is given as a function of time \( t \) (measured in hours):

   \[ v(t) = \begin{cases} 
   50t^{3/2}, & \text{for } 0 \leq t \leq 1 \\
   50, & \text{for } t > 1 
   \end{cases} \]

   What distance has the car covered after 2 hours?

   - 60 miles.
   - 70 miles. A
   - 100 miles.
   - 150 miles.
3. The improper integral
\[ \int_{1}^{\infty} \frac{1}{x^3} \, dx \]
- does not exist.
- exists and equals \( \frac{1}{4} \).
- exists and equals \( \frac{1}{2} \). A
- exists and equals 1.

4. The definite integral
\[ \int_{1}^{2} \frac{\ln x}{x} \, dx \]
equals:
- \( \ln \frac{2}{2} \).
- 2 \ln 2.
- \( \ln \frac{4}{2} \).
- \( \frac{(\ln 2)^2}{2} \). A

5. Find
\[ \int \cos x \sin^2 x \, dx \]
- \( -\frac{\sin^3 x}{3} + c. \)
- \( \frac{\sin^3 x}{3} + c. A \)
- \( -\frac{\cos^3 x}{3} + c. \)
- \( \frac{\cos^3 x}{3} + c. \)

6. Find
\[ \int_{1}^{2} x^2 \ln x \, dx \]
- \( \frac{8}{3} \ln 2. \)
- \( \frac{8}{3} \ln 2 - \frac{7}{9}. A \)
- 2 \ln 2.
- 4 \ln 2.

7. Determine the following indefinite integral\(^1\):
\[ \int \tan x \, dx. \]

**Sol:** See example 2 on p. 23 in the lecture notes.

\(^1\)Recall that \( \tan x = \frac{\sin x}{\cos x} \)
8. Let \( u(t) \) be the density of a growing bacterial population, measured in g/l. If the rate of change of the density with respect to time \( t \) (measured in minutes) is given by:

\[
(1 + t)^{\frac{3}{2}},
\]

and assuming that the initial density at time \( t = 0 \) is \( u(0) = 1 \) g/l, find the density at time \( t = 1 \) minute.

**Sol:** By FTC: \( u(1) - u(0) = \int_0^1 (1 + t)^{3/2} \, dt \). By substitution \( u = 1 + t \), so \( du = dt \):

\[
\int (1 + t)^{3/2} \, dt = \int u^{3/2} \, du = 2u^{5/2}/5 + c = 2(1 + t)^{5/2}/2 + c.
\]

Thus, using the FTC, \( u(1) = 1 + (2/5)(2^{5/2} - 1) = \frac{3+2^{7/2}}{5} \).