Midterm 2 Solution Key MTH 228*

November 17, 2017

Name:

This is a **closed book** exam and the use of formula sheets or calculators is **not** permitted.

The first 6 problems are multiple choice problems, **each worth 5 points**. Please circle the right answer (there is exactly one for each problem). The last 2 problems require that you show all work, and will receive partial credit. They are worth **15 points each**.

1. Let
$$f(x) = \frac{1}{2} + x^3,\ 0 \leq x \leq 4.$$
We wish to approximate the area under the graph of f and above the horizontal axis, when x varies between 0 and 4. To do so, we partition $[0, 4]$ in 4 subintervals of length 1, and use the function value in the **left** endpoint of each subinterval. The approximate area is:

   - 38. **A**
   - 66.
   - 102.
   - $\frac{129}{2}$.

2. A car's velocity (measured in miles per hour) is given as a function of time t (measured in hours):
$$v(t) = \begin{cases} 50t^{3/2}, & \text{for } 0 \leq t \leq 1 \\ 50, & \text{for } t > 1 \end{cases}$$
What distance has the car covered after 2 hours?

   - 60 miles.
   - 70 miles. **A**
   - 100 miles.
   - 150 miles.

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3. The improper integral

$$\int_1^\infty \frac{1}{x^3} dx$$

- does not exist.
- exists and equals $\frac{1}{4}$.
- exists and equals $\frac{1}{2}$. **A**
- exists and equals 1.

4. The definite integral

$$\int_1^2 \frac{\ln x}{x} dx$$

equals:

- $\frac{\ln 2}{2}$.
- $2 \ln 2$.
- $\frac{\ln 4}{2}$.
- $\frac{(\ln 2)^2}{2}$. **A**

5. Find

$$\int \cos x \sin^2 x dx$$

- $-\frac{\sin^3 x}{3} + c$.
- $\frac{\sin^3 x}{3} + c$. **A**
- $-\frac{\cos^3 x}{3} + c$.
- $\frac{\cos^3 x}{3} + c$.

6. Find

$$\int_1^2 x^2 \ln x dx$$

- $\frac{8}{3} \ln 2$.
- $\frac{8}{3} \ln 2 - \frac{7}{9}$. **A**
- $2 \ln 2$.
- $4 \ln 2$.

7. Determine the following indefinite integral[1]:

$$\int \tan x dx.$$

Sol: See example 2 on p. 23 in the lecture notes.

[1] Recall that $\tan x = \frac{\sin x}{\cos x}$

8. Let $u(t)$ be the density of a growing bacterial population, measured in g/l. If the rate of change of the density with respect to time t (measured in minutes) is given by:

$$(1+t)^{\frac{3}{2}},$$

and assuming that the initial density at time $t = 0$ is $u(0) = 1\ g/l$, find the density at time $t = 1$ minute.

Sol: By FTC: $u(1) - u(0) = \int_0^1 (1+t)^{3/2} dt$. By substitution $u = 1 + t$, so $du = dt$: $\int (1+t)^{3/2} dt = \int u^{3/2} du = 2u^{5/2}/5 + c = 2(1+t)^{5/2}/2 + c$. Thus, using the FTC, $u(1) = 1 + (2/5)(2^{5/2} - 1) = \frac{3+2^{7/2}}{5}$.