Sample questions on Parts III and IV for Final Exam MTH 228*

December 4, 2017

Notes: As mentioned during the last lecture, this exam will have a similar format as the 2 midterms. There will be about 10 multiple choice questions and about 4 open problems which will receive partial credit. The exam will be cumulative and will also include questions about Parts I and II. For sample problems on those Parts, look at the sample midterms and the midterms which have been posted online with their solution key. The final exam will be a **closed book** exam and the use of calculators will **not** be permitted. You may however use a one-sided formula sheet after it has been approved by the instructor at the beginning of the exam.

1. (**Note: There were 2 typos in the original statement of this problem: The pdf should have been as corrected below, and the 2nd answer should have been $k = 1/4$, not $k = 0$**)
   Let X be a continuous random variable with range $S = [-2, 2]$ and probability density function:
   $$f(x) = k(2 - |x|) \text{ for } x \text{ in } S.$$
   Then there must hold that:
   - $k = -1$.
   - $k = 1/4$. **A**
   - $k = 1/2$.
   - $k = 1$.

2. Let X be a uniform random variable with $S = [0, 3]$. The probability that X belongs to $[1, 2]$ equals:
   - $1/4$.
   - $1/3$. **A**
   - $1/2$.
   - $2/3$.

3. The inverse of the matrix
   $$T = \begin{pmatrix} -2 & 2 \\ 3 & -3 \end{pmatrix}$$

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- does not exist. **A**
- exists and equals $\begin{pmatrix} -2 & -2 \\ -3 & -3 \end{pmatrix}$.
- exists and equals $\begin{pmatrix} -3 & 2 \\ 3 & -2 \end{pmatrix}$.
- exists and equals $\begin{pmatrix} -3 & -2 \\ -3 & -2 \end{pmatrix}$.

4. Given are the following matrices:

$$A = \begin{pmatrix} 1 & 1 \\ 2 & -3 \end{pmatrix}, \; B = \begin{pmatrix} 0 & -2 \\ 1 & 1 \\ 4 & 0 \end{pmatrix}, \; C = \begin{pmatrix} 2 & -2 & 1 \\ 1 & 0 & 1 \end{pmatrix}.$$

Calculate

$$-2A + 4B, \; AB, \; CB, \; \text{and } A^{-1},$$

provided these matrices exist. If they do not exist, explain why.

$-2A+4B$ does not exist because $-2A$ and $4B$ do not have the same size (e.g. 2 and 3 rows respectively)

AB does not exist because A has 2 columns but B has 3 rows.

$CB = \begin{pmatrix} 2 & -6 \\ 4 & -2 \end{pmatrix}$.

$A^{-1} = \begin{pmatrix} \frac{3}{5} & \frac{1}{5} \\ \frac{2}{5} & -\frac{1}{5} \end{pmatrix}$.

5. - Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 2 \end{pmatrix}.$$

Eigenvalues: $\lambda_1 = 1$ and $\lambda_2 = 2$ with corresponding eigenvectors $v_1 = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ and $v_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

- Calculate A^{101}.

$A^{101} = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 2^{101} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2(2^{101}-1) & 2^{101} \end{pmatrix}$.

6. Amantha has a long shift ahead at one of the checkout registers of the Thrifty Town, and she wants to know how busy she will be. The store manager told the store employees that the waiting time for the next customer at the checkout registers is an exponential random variable with $\lambda = 2$ per minute.

- Calculate the probability that Amantha has to wait less than 5 minutes for the next customer to show up.

$$P(T < 5) = \int_0^5 2\,\mathrm{e}^{-2x}\,dx = 1 - \mathrm{e}^{-10}.$$

- Calculate the probability that she has to wait at least 10 minutes for the next customer to show up.

$$P(T < 5) = \int_{10}^{\infty} 2\,\mathrm{e}^{-2x}\,dx = \mathrm{e}^{-20}\,.$$