Sample Midterm 1 MTH 228*

October 11, 2017

Name:

This is a **closed book** exam and the use of formula sheets or calculators is **not** permitted.

The first 6 problems are multiple choice problems, **each worth 5 points**. Please circle the right answer (there is exactly one for each problem). The last 2 problems require that you show all work, and will receive partial credit. They are worth **15 points each**.

1. The derivative of the function
$$\mathrm{e}^{\cos x}$$
is
   - $\mathrm{e}^{-\sin x}$.
   - $-\sin x\, \mathrm{e}^{\cos x}$
   - $\cos x\, \mathrm{e}^{\cos x}$
   - $-\sin x\, \mathrm{e}^{-\sin x}$

2. The derivative of the function [1]
$$\tan x, \quad \text{where } x \neq \frac{\pi}{2} + k\pi \text{ and } k \text{ is any integer,}$$
is
   - $\cot x$
   - $-\cot x$
   - $\frac{\sin^2 x - \cos^2 x}{\cos^2 x}$
   - $\frac{1}{\cos^2 x}$

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[1] Recall that $\tan x = \frac{\sin x}{\cos x}$ and $\cot x = \frac{\cos x}{\sin x}$.

3. The derivative of the function

$$e^{2x}\cos(3x)$$

is

- $e^{2x}\,(2\cos(3x) - 3\sin(3x))$
- $-6\,e^{2x}\sin(3x)$
- $e^{2x}\,(2\cos(3x) + 3\sin(3x))$
- $e^{2x}\,(\cos(3x) - \sin(3x))$

4. The function

$$f(x) = \frac{1}{3}x^3 - x,\ -2 \le x \le 2$$

- has a local minimum at $x = 1$ and a local minimum at $x = -1$.
- has a local maximum at $x = 1$ and a local maximum at $x = -1$.
- has a local minimum at $x = 1$ and a local maximum at $x = -1$.
- has a local maximum at $x = 1$ and a local minimum at $x = -1$.

5. The function

$$f(x) = \frac{1}{3}x^3 - x,\ -2 \le x \le 2$$

- has a unique global maximum at $x = -1$ and a unique global minimum at $x = 1$.
- has two global maxima at $x = -1$ and at $x = 2$, and two global minima at $x = -2$ and at $x = 1$.
- has no global maxima or global minima.
- has a unique global maximum at $x = 2$ and a unique global minimum at $x = -2$.

6. The function

$$f(x) = \cos x,\ 0 \le x \le \pi,$$

- has no inflection points.
- has an inflection point at $x = \frac{\pi}{4}$.
- has an inflection point at $x = \frac{\pi}{2}$.
- has two inflection points at $x = 0$ and at $x = \pi$.

7. Show that the following logistic "growth" curve:

$$N(t) = \frac{2}{2 - e^{-t}},\quad t \ge 0$$

is always decreasing.

8. An ornithologist wants to determine the optimal clutch size of an exotic bird, knowing that clutch fitness, as a function of the number of eggs N is given by:

$$F(N) = N\,e^{-N/4}$$

Help her determine the optimal clutch size.