Homework assignment 1[*]

January 6, 2006

1. Give 4 examples illustrating the 4 types of linearized stability that may occur as we discussed in class.

2. In class we showed the outcome of a population under contest competition and under scramble competition by constructing the cobweb in case the basic reproductive ratio $R_0 > 1$. Discuss in both cases what happens if $R_0 \in (0, 1]$. Do you notice any differences in the outcomes? Also discuss linearized stability for all fixed points.

3. Suppose a population is governed by

   $$N_{t+1} = f(N_t),$$

   where $f(0) = 0$. Assume that $f : \mathbb{R}_+ \to \mathbb{R}_+$ is non-decreasing (that is: $x \leq y$ implies that $f(x) \leq f(y)$), and that there is some $M > 0$ such that $f(N_t) \leq M$ for all N_t.

   Prove that every solution sequence N_t converges as $t \to \infty$.[1] (Convince yourself first by constructing some cobwebs for non-decreasing f's). Notice that contest competition is a special case.

   Suppose that f is continuously differentiable and that the fixed points are isolated and that f' is never equal to 1 at each fixed point. Show (geometrically) that the fixed points alternate bewteen monotonically stable and monotonically unstable.

4. Verify the occurrence of a transcritical bifurcation for Hassel's equation at the bifurcation point $(R_0, x) = (1, 0)$.

[*]MAP 4484/5489; Instructor: Patrick De Leenheer.

[1]It may be helpful to review the chapter on *Sequences* as it is taught in your Calc course.