Homework assignment 1*

Due date: Monday February 12, 2007

1. **Laguerre's equation**

   The Laguerre equation is:

   $$xy'' + (1-x)y' + ny = 0,$$

   where n is a non-negative integer. Show that for every n, Laguerre's equation has a polynomial solution of degree n, and determine these polynomials for $n = 0, 1, 2$ and 3.

2. **Method of Frobenius I**

   Find the first 3 terms of the series expansion about $x = 0$ of 2 linearly independent solutions to

   $$x^2y'' - x^2y' + (x^2 - 2)y = 0$$

3. **Method of Frobenius II**

   Determine the form of a series expansion about $x = 0$ of 2 linearly independent solutions to

   $$xy'' - sy' + x^3y = 0,$$

   where s is an arbitrary real number. Don't determine the coefficients of the series. Your answer should depend on the value of s.

4. **Property of the Gaussian hypergeometric function.**

   Denoting the Gaussian hypergeometric function by $F(\alpha, \beta, \gamma; x)$, show that

   $$\ln(1+x) = xF(1, 1, 2; -x).$$

5. **Properties of Bessel functions.**

   Denoting the Bessel function of the first kind of order $\nu > 0$ by $J_\nu(x)$, show that the following properties hold:

   $$\frac{d}{dx}\left(x^{-\nu}J_\nu(x)\right) = -x^{-\nu}J_{\nu+1}(x) \text{ and } J_{\nu+1}(x) = \frac{2\nu}{x}J_\nu(x) - J_{\nu-1}(x).$$

*MAP 4305; Instructor: Patrick De Leenheer.