Homework assignment 2*

Due date: March 23. 2007.

1. Determine all real eigenvalues and corresponding eigenfunctions of

$$y'' + \lambda y = 0, \;\; y(0) = 0, \;\; y(\pi) + y'(\pi) = 0.$$

2. Consider the linear operator

$$L[y] = y^{(4)},$$

defined on the set of functions having continuous derivatives up to order 4, that satisfy the following boundary conditions:

$$y(a) = y'(a) = y(b) = y'(b) = 0.$$

Show that L is self-adjoint, ie that for all y_1 and y_2 in the domain of L, we have that:

$$(y_1, L[y_2]) = (L[y_1], y_2).$$

Use this to prove that eigenfunctions of the problem:

$$L[y] + \lambda y = 0, \;\; y(a) = y'(a) = y(b) = y'(b) = 0$$

corresponding to distinct eigenvalues are orthogonal, ie that

$$(y_1, y_2) = 0,$$

whenever y_1 and y_2 are eigenfunctions corresponding to distinct eigenvalues.

3. Consider the following boundary value problem:

$$y'' + y = \sin 2x, \;\; y(0) = y(2\pi), \;\; y'(0) = y'(2\pi).$$

- Using the Fredholm alternative, determine whether or not this problem has solutions.
- If there are solutions, determine them.

4. Consider the following nonhomogeneous boundary value problem:

$$y'' + y = f(x), \;\; y(0) = y(1) = 0,$$

where $f : [0, 1] \to \mathbb{R}$ is given by

$$f(x) = x(1 - x).$$

Determine a solution of this problem, written as a formal series expansion using an orthonormal system of eigenfunctions of the associated homogeneous problem.

*MAP 4305; Instructor: Patrick De Leenheer.