

Practice Exam 1: MAP 4305*

1. Does

$$5xy'' + 4(1 - x^2)y' + y = 0, \quad x > 0,$$

have a solution which is bounded near zero? Notice that to answer this question, you only need to consider the indicial equation.

2. Determine the form of a series expansion about $x = 0$ of 2 linearly independent solutions to:

$$x^2y'' - xy' + (1 - x^2)y = 0, \quad x > 0.$$

Do not find a recursion formula for the coefficients.

3. Let $J_\nu(x)$ be the Bessel function of the first kind of order $\nu \geq 0$:

$$J_\nu(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n! \Gamma(1 + \nu + n)} \left(\frac{x}{2}\right)^{2n+\nu}.$$

Prove that

$$J_{\nu+1}(x) = J_{\nu-1}(x) - 2J'_\nu(x).$$

4. Find the first three non-zero terms in a series expansion about $x = 0$ of 2 linearly independent solutions to:

$$3xy'' + (2 - x)y' - y = 0, \quad x > 0.$$