# Practice Exam 2: MAP 4305*

1. Find the **positive** eigenvalues and eigenfunctions of the following eigenvalue problem:

$$y'' + \lambda y = 0, \ \ y(0) + y'(0) = y(\pi) = 0.$$

2. Find conditions on $f$ so that the following non-homogeneous BV problem has a solution:

$$y'' - y' + 3y = f, \ \ y(0) = y(\pi) = 0$$

3. Consider the linear operator

$$L[y] = y^{(4)},$$

defined on the set of functions having continuous derivatives up to order 4, that satisfy the following periodic boundary conditions:

$$y^{(i)}(a) = y^{(i)}(b), \ \ i = 0, 1, 2, 3.$$

Show that $L$ is self-adjoint with respect to the usual inner product for functions on $(a, b)$.

4. Using the method of eigenfunction expansion, solve the following non-homogeneous BV problem:

$$y'' + 9y = 1 + \cos(x), \ \ y'(0) = y'(\pi) = 0,$$

given that the eigenvalues and eigenfuctions of $y'' + \lambda y = 0, \ \ y'(0) = y'(\pi) = 0$ are $\lambda_n = n^2$, $n = 0, 1, \ldots$, and $\phi_n(x) = c_n \cos(nx)$ where the $c_n$ are arbitrary nonzero constants.

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