Homework assignment 4*

Due date: Monday April 21, 2008.

1. (# 9.5.5) Inside a circle of radius a, consider Poisson's equation:

$$\nabla^2 u = f(x), u(a,\theta) = h_1(\theta) \text{ for } \theta \in (0,\pi), \text{ and } u_r(a,\theta) = h_2(\theta) \text{ for } \theta \in (-\pi, 0).$$

   Represent the solution $u(r,\theta)$ in terms of the Green's function (assumed to be known). Write down the problem to which this Green's function is a solution, but do not solve that problem.

2. (# 9.5.14) Using the method of images, solve

$$\nabla^2 G = \delta(x - x_0),$$

   in the first quadrant $x > 0$ and $y > 0$ with $G = 0$ on the boundary.

3. (# 9.5.19) Determine the Green's function inside a semi-disk ($0 < r < a$ and $0 < \theta < \pi$)

$$\nabla^2 G = \delta(x - x_0),$$

   with $G = 0$ on the boundary.

4. (# 12.2.4) Solve using the method of characteristics:

$$\frac{\partial w}{\partial t} + c\frac{\partial w}{\partial x} = 0 \ (c > 0),$$

   on the first quadrant $x > 0$ and $t > 0$ if

$$w(x,0) = f(x), \ \ x > 0 \text{ and } w(0,t) = h(t), \ \ t > 0.$$

5. (# 12.2.5 (a) and (d)) Solve using the method of characteristics:

$$\frac{\partial w}{\partial t} + c\frac{\partial w}{\partial x} = \mathrm{e}^{2x}, \ \ w(x,0) = f(x),$$

   and

$$\frac{\partial w}{\partial t} + 3t\frac{\partial w}{\partial x} = w, \ \ w(x,0) = f(x),$$

6. (# 12.6.6 (a)) Consider the following traffic flow problem:

$$\frac{\partial \rho}{\partial t} + c(\rho)\frac{\partial \rho}{\partial x} = 0.$$

   Assume that $u(\rho) = u_{\max}(1 - \rho/\rho_{\max})$ and $c(\rho) = d/d\rho(\rho u(\rho))$. Solve for $\rho(x,t)$ if

$$\rho(x,0) = \begin{cases} \rho_{\max}, & x < 0 \\ 0, & x > 0 \end{cases}.$$

   This initial condition corresponds to an infinite line of traffic stopped at a red light at $x = 0$ which is started by the light turning green.

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