8.8 #24 $\quad x^{\nu}J_{\nu}$ satisfies $xy'' + (1-2\nu)y' + xy = 0$

$$x(x^{\nu}J_{\nu})'' + (1-2\nu)(x^{\nu}J_{\nu})' + xJ_{\nu} = x\frac{d}{dx}\left(\nu x^{\nu-1}J_{\nu} + x^{\nu}J_{\nu}'\right) + (1-2\nu)\left(\nu x^{\nu-1}J_{\nu} + x^{\nu}J_{\nu}'\right) + x^{\nu+1}J_{\nu}$$

$$= x\left(\nu(\nu-1)x^{\nu-2}J_{\nu} + \nu x^{\nu-1}J_{\nu}' + \nu x^{\nu-1}J_{\nu}' + x^{\nu}J_{\nu}''\right) + (1-2\nu)\left(\nu x^{\nu-1}J_{\nu} + x^{\nu}J_{\nu}'\right) + x^{\nu+1}J_{\nu}$$

$$= x^{\nu+1}J_{\nu}'' + \left(\cancel{2\nu x^{\nu}} + (1-\cancel{2\nu})x^{\nu}\right)J_{\nu}' + \left(\nu(\nu-\cancel{1})x^{\nu-1} + (\cancel{1}-2\nu)\nu x^{\nu-1} + x^{\nu+1}\right)J_{\nu}$$

$$= x^{\nu+1}J_{\nu}'' + x^{\nu}J_{\nu}' + (x^2-\nu^2)x^{\nu-1}J_{\nu} = x^{\nu-1}\left[x^2J_{\nu}'' + xJ_{\nu}' + (x^2-\nu^2)J_{\nu}\right] = 0$$

since J_{ν} satisfies Bessel eqn of order ν

Sol. of $xy'' - 2y' + xy = 0$ is $(\nu = \frac{3}{2})$. $x^{\frac{3}{2}}J_{\frac{3}{2}}(x)$.