1. Royden, p. 64, # 14.

2. In this problem, let \( \mu \) be a measure on a \( \sigma \)-algebra \( \mathcal{A} \) (actually, additive function on a ring of sets is enough).

   (a) Prove that if \( A_1, A_2 \) belong to \( \mathcal{A} \) then
   \[ \mu(A_1 \cup A_2) + \mu(A_1 \cap A_2) = \mu(A_1) + \mu(A_2). \]
   (Note, you have to worry about infinities.)

   (b) Suppose \( A_i \) for \( i = 1, \ldots, n \) belong to \( \mathcal{A} \) and have finite measure. Prove that
   \[ \mu(\bigcup_{i=1}^{n} A_i) = \sum_{i=1}^{n} \mu(A_i) - \sum_{i<j} \mu(A_i \cap A_j) + \sum_{i<j<k} \mu(A_i \cap A_j \cap A_k) + \cdots + (-1)^{n-1} \mu(\bigcap_{i=1}^{n} A_i). \]
   (For \( \mu \) counting measure, this is called the generalized inclusion-exclusion principle.)

3. Let \( f : \mathbb{R} \to \mathbb{R} \) satisfy the condition that there is a \( K > 0 \) such that \( |f(x) - f(y)| \leq K|x - y| \) for all \( x, y \in \mathbb{R} \). Let \( m^* \) be Lebesgue outer measure on \( \mathbb{R} \), and let \( E \) be a set with \( m^*(E) = 0 \). Prove that \( m^*(f(E)) = 0 \). Is it necessarily true that \( m^*(f^{-1}(E)) = 0 \)?
Now answer the same questions with Hausdorff \( d_\rho \) outer measure.