1. Prove that if $E$ is a Lebesgue measurable set with positive measure, then the set $E - E = \{x : x = y - z, \ y \in E, \ z \in E\}$ contains an open interval around 0. [Hint: Use the $m(E \cap I) > \alpha m(I)$ problem from the midterm.]

2. Royden, p. 70, # 19.


4. Let $X, d$ be a metric space, $\mu^*$ an outer measure on $X$, and $\mathcal{M}_{\mu^*}$ the $\sigma$-algebra of $\mu^*$-measurable sets. Prove that every continuous real-valued function on $X$ is measurable, if and only if $\mu^*$ is a metric outer measure. [Hint: If $F$ is a closed set, then the function $g(x) = d(x, F)$ is continuous.]