1. Royden, p. 122, # 7. (You may want to do 7b first.)

2. Suppose that $1 \leq p < \infty$, $f \in L^p(R)$, and that $f$ is uniformly continuous. Prove that 
   \[ \lim_{x \to \pm\infty} f(x) = 0. \]
   Show that this need not be the case if $f$ is uniformly continuous is weakened to continuous.

3. Let $E$ be a measurable set in $R$, $0 < p_0 < p_1 < \infty$, and suppose that $f \in L^{p_0}(E) \cap L^{p_1}(E)$. If $p_0 < r < p_1$ prove that $f \in L^r(E)$ by considering the sets where $|f| \leq 1$ and $|f| > 1$.

4. Suppose $0 < p_0 < p_1 < \infty$. Using functions of the form $x^{-\alpha} \log x|^{\beta}$ show for each case below that there exists a function $f$ on $(0, \infty)$ such that $f \in L^p$ if and only if
   
   (a) $p_0 < p < p_1$
   
   (b) $p_0 \leq p \leq p_1$
   
   (c) $p = p_0$

5. Suppose that $E$ is a measurable set with measure equal to 1. Prove that if $f$ and $g$ are positive measurable functions such that $fg \geq 1$ everywhere on $E$, then
   \[ \int_E f \int_E g \geq 1. \]