1. Royden, problem 16 on page 126.

2. Show that there is an uncountable family $\mathcal{F}$ of functions in $L^\infty(R)$ such that for all $f, g \in \mathcal{F}$ with $f \neq g$, $\|f - g\| \geq 1$.

3. Let $\mathcal{M}$ be the set of equivalence classes of almost everywhere finite measurable functions (under equality a.e.) on a measurable set $E$ of finite measure. Define $\rho$ by

$$\rho(f, g) = \int_E \frac{|f - g|}{1 + |f - g|}.$$ 

Prove that the integral is convergent and defines a metric on $\mathcal{M}$.

4. Royden, problem 4 on page 143.

5. Let $S$ the set of infinite sequences of natural numbers. Define $\rho$ on $S \times S$ by $\rho(\{a_i\}, \{b_i\}) = 0$ when $a_i = b_i$ for all $i$ and is equal to $\frac{1}{n}$ if $a_i = b_i$ for $i < n$ but $a_n \neq b_n$. Prove that $\rho$ is a metric. Define $L : S \rightarrow S$ by $L(\{a_i\}) = b_i$ with $b_i = a_{i+1}$. Show whether or not $L$ is continuous.