1. Royden, problem 9 on page 145.

2. Let $Lip(M)$ be the set of Lipschitz continuous functions on $[0, 1]$ with Lipschitz constant $C$ with $C \leq M$ ($M > 0$). Show that $Lip(M)$ is a closed subset of $C([0, 1])$, $\| \cdot \|_{\infty}$.

3. Suppose that $f$ is a measurable function on $\mathbb{R}$ which is integrable over every bounded interval. Find necessary and sufficient conditions on $f$ so that $\rho(x, y) = \int_{[x,y]} f$ defines a metric on $\mathbb{R}$ – here we suppose that $x \leq y$ in defining the interval, but impose that $\rho(x, y) = \rho(y, x)$. Then find necessary and sufficient conditions on $f$ for $\mathbb{R}$ to be complete with respect to this metric.

4. Let $\mathcal{M}$ be the set of equivalence classes of almost everywhere finite measurable functions (under equality a.e.) on a measurable set $E$ of finite measure. Let $\rho$ be the metric

$$\rho(f,g) = \int_E \frac{|f - g|}{1 + |f - g|}.$$ 

Prove that $f_n$ converges in measure to $f$ if and only if $f_n$ converges to $f$ with respect to $\rho$. Conclude that $\mathcal{M}$ is complete.