Unless a problem states otherwise, you may use software to assist your calculations. If you do use some package, you must also include a printout of your computer work.


2. Consider the system of differential equations

\[ \begin{align*}
    x' &= y \\
    y' &= -cy - rx(1 - x).
\end{align*} \]

(a) Find the critical (equilibrium) points, and classify their local stability. (This will depend on \( r \) and \( c \).)

(b) Assume \( c > 0 \). Let \( H(x, y) = r/2 x^2 + 1/2 y^2 - r/3 x^3 \). Compute the directional derivative of \( H \) in the direction \( F, \nabla H \cdot F \), (where \( F \) is the right hand side of the system). Show that a trajectory starting in the region \( \{(x, y) : H(x, y) < H(1, 0)\} \) remains in this region.

3. The following equation arose in relativity theory.

\[ \frac{d^2 u}{d\theta^2} + u = \alpha + \epsilon u^2 \]

It is assumed that \( \alpha \) and \( \epsilon \) are positive constants.

(a) Rewrite this as a system using \( x = u \) and \( y = u' \), where the derivatives are with respect to \( \theta \).

(b) Find the equilibrium (critical) points, and classify the linearization at each critical point.

(c) Classify the equilibrium points of the non-linear system.