

3. Let $w = f(z) = z^i$ be defined using the principal branch of the logarithm. Describe the image in the $w$-plane of circles centered at 0 in the $z$-plane and rays from 0 in the $z$-plane (omitting points $z$ on the negative real axis, of course).

4. Let $P : S \rightarrow \mathbb{C}_\infty$ be stereographic projection. For any map $G$ of $S$ to itself there is induced a map $g$ of $\mathbb{C}_\infty$ to itself by $g = P \circ G \circ P^{-1}$.

(a) Let $R_\theta$ be the rotation of $\mathbb{R}^3$ whose standard matrix is

$$
\begin{bmatrix}
\cos \theta & -\sin \theta & 0 \\
\sin \theta & \cos \theta & 0 \\
0 & 0 & 1
\end{bmatrix}
$$

that the induced map of $\mathbb{C}_\infty$ is given by a Möbius transformation.

(b) Let $T$ be the rotation of $\mathbb{R}^3$ which maps $e_1$ to $e_3$, $e_3$ to $-e_1$, and fixes $e_2$. Show that the induced map of $\mathbb{C}_\infty$ is given by a Möbius transformation.

(c) It can be shown that $R_\theta$ for $\theta \in [0, 2\pi)$ and $T$ generate $SO(3)$, the group of rotations of $\mathbb{R}^3$. You can conclude that every rotation of $S$ induces a Möbius transformation. Show that there exists a Möbius transformation which is not induced by a rotation. (You may assume that every rotation has an eigenvector with eigenvalue 1.)