1. Conway, Chapter 6, section 2, #2 and #8.

2. Suppose that $f$ is continuous on the closed unit disk and analytic on the open unit disk, $f(0) = \frac{1 + \sqrt{3}i}{2}$ and $|f(z)| > \frac{\sqrt{5}}{2}$ when $|z| = 1$. Prove that $f(a) = 0$ for some $a$ in the unit disk.

3. Suppose that $f$ is a one-to-one analytic mapping of the open unit disk onto the open unit square, $\{z : -1 < \text{Re} \, z < 1, \ -1 < \text{Im} \, z < 1\}$. Prove that $f(iz) = if(z)$ for all $z$ in the unit disk. [Hint: Let $g = if(z)$ and consider $f^{-1} \circ g$.]

4. Let $f$ be an entire function, and suppose that there exist positive numbers $K$ and $m$ such that

$$\text{Re} \, f(z) \leq K + m \log(1 + |z|) \quad \text{for all } z \in \mathbb{C}.$$ 

Prove that $f$ is constant.

5. Suppose that $f$ is analytic on the closed disk of radius 1. For every $0 < s < 1$ prove that there exists a constant $C$ (which may depend on $s$) such that

$$\|f\|_{L^\infty(B_s)} \leq C \|f\|_{L^2(D)}.$$ 

Here $D$ is the unit disk, $B_s$ is the disk of radius $s$ centered at the origin, $\|f\|_{L^\infty}$ is the supremum norm, and $\|f\|_{L^2}^2 = \int_D |f|^2 \, dx \, dy$. [Hint: For $z_0 \in B_s$, use the mean value property, integration in polar coordinates over a suitable disk on annulus, and the Cauchy-Schwarz inequality.]