ACTIVITIES

1 An Overview

This is the annual report for the second year of my NSF grant, DUE-0410641. It covers the time period from September 2005 to September 2006. The purpose of the grant is to adapt Ken Bogart’s successful project, “Teaching Introductory Combinatorics by Guided Group Discovery”, for a required discrete mathematics course for math majors. The prototype course here at Oregon State is taught once a year.

The notes are now in their third adaptation since I first used Ken’s notes in Fall 2003. After my colleague John Lee taught the course in Fall 2005, he had many suggestions which I incorporated into the third adaptation. Adapting the notes has taken significantly more time and effort than I anticipated in the original proposal, but it has also been satisfying to see the student response and to work with John on the continuing adaptation. Section 2 contains information on the adaptations, and a comparison of Ken’s notes with the current version is included in Section 2.1. In addition to the notes, an accompanying instructor handbook is in process. John and I are working on this together, and more information is given in Section 2.2.

Section 3 of this report gives an overview of the various implementations of Ken’s guided discovery method in our classes, including some discussion of difficulties which have been encountered. Section 4 concentrates on the perceived longterm effect of this class on our majors. This is a very short synopsis of discussions that John and I have had with colleagues. We intend to expand this section for publication.

My dissemination to date has been: designing posters to present at two January meetings; writing a chapter for inclusion in the MAA Notes volume on the teaching of discrete mathematics; construction of a Web page; and some e-mail correspondence with people who have expressed interest in the project. Two other likely venues for presentation are the annual meetings of the Oregon Academy of Science and of the PNW section of the MAA, but unfortunately last year both of those meetings were held on dates when I had other professional commitments. John or I will make a presentation at these meetings this year.

In September Rosa Orellana (Dartmouth) agreed to serve as a consultant on the project. We’ve arranged that she’ll analyze the current notes and class organization from the perspective of someone who was very involved in the original project. I’ve asked her to identify places which have strayed too far from the spirit of guided discovery, trying to be very vague so as not to influence her perspective. Rosa will provide the needed perspective of someone who knows Ken’s notes well and who has taught from the original notes several times.

The size of our class at OSU is steadily increasing. This term the enrollment was 29, and the class was full before registration ended in the spring. Our department is willing to begin offering two sections every Fall, and we expect the enrollment to stabilize to about 20 students in each class. It is hoped that this arrangement will promote the longterm health of the course. In addition, John and I have suggested that faculty members be encouraged to teach the course for at least two consecutive
years. We think this arrangement will serve as a built-in mentoring of faculty who are new to the course, but we also make this recommendation because we think a promise of two years would be a sign that the course instructor has a solid commitment to helping the evolution of the course at OSU.

2 The Adapted Notes—An overview

In this section I give a summary of the salient features of the adaptation of the notes. Because it summarizes several years work, there is some overlap with last year’s report. My Website has links to a copy of the three adapted versions as well as an additional copy of the 2006 adaptation which is annotated in red to indicate significant changes from the 2005 edition. Ken and I agreed on the changes made in the first adaptation. For the second adaptation, Ken and I discussed the Fall 2004 course as I was teaching it and we had a several-hour discussion at the January 2005 meetings. However, most of the changes in the Fall 2005 version were made after Ken’s death in March 2005. A table comparing the placement of topics in the current adaptation with Ken’s notes is given in Section 2.1.

A substantial change was the incorporation of much of the material in Ken’s appendices into the main body of the notes. This was done because we cannot assume our students have more than a superficial acquaintance with this material (principally functions, equivalence relations, and induction). The appendix on exponential generating functions (prepared for graduate classes by Vic Reiner) has been removed.

Ken’s first chapter was quite long, and after material from Ken’s appendices was included the length became daunting. Why do I say daunting? In the first chapter the student is introduced to guided discovery—really to the habit of thinking for themselves—as well as to counting techniques which are probably new to them. I have reduced the number of different techniques and ideas in each of the first few chapters in order that the students feel more of a sense of accomplishment, which in turn allows them to become more comfortable with what’s expected of them. The first chapter also now ends with a problem sequence which is designed to personalize their understanding of guided discovery especially from the viewpoints of how to approach problems and the importance of regularly summarizing what they’ve learned.

Many of our students don’t have sufficient mathematical maturity to ferret out definitions and theorems from the problem sequences. Because of this, I’ve included more summarizing material in the notes, more exposition of where the notes are headed and where they’ve been, and more leading comments such as “consider small/specific cases”. These are things I’d like Rosa to evaluate—for instance, has this additional material negatively affected guided discovery? Barbara Edwards has included a discussion of some students’ reaction to the amount of exposition in the 2005 adaptation. For example, she notes that “The students find the materials easy to read but rather terse. Most of them seem to adjust at least somewhat to this as the course progresses.” To me that comment indicates the amount of written exposition is about right.

In the first adaptation some of the more esoteric counting was moved to supplementary sections because Ken and I agreed these topics could properly be viewed as
peripheral to our course, a more general course in discrete mathematics rather than the original one in enumerative combinatorics. We decided these problem sequences would be retained in supplementary sections to be worked by groups who were progressing more quickly through the main notes. Those students could then take an enriching detour from the main notes, while other groups could catch up. (This also has the advantage that most of the students would be at roughly the same place most of the time.) In Fall 2004, I encouraged one group of three students to work on supplementary problems, but in Fall 2005 none of the groups did much supplementary work. In an effort to try to get more students to do more problems, for the 2006 version John and I decided most of these problem sequences should be returned to the main notes and labelled as optional. With this placement it is hoped that the stronger students will now consider them to be less serious deviations from the main course and will be more likely to try them. (We will include a discussion of what we have tried in this regard as well as what has worked in the instructor handbook.)

When compared with Ken’s notes, the current version of our notes has some significant changes in mathematical approach. The first chapter now includes a problem sequence which more thoroughly examines the Sum Principle and its logical relation to the Product Principle. It also develops the Sum Principle as a special case of the Principle of Inclusion/Exclusion. The sequence expects a higher level of thinking, including having the groups explore the notion of partition in anticipation of equivalence relations.

As I indicated in last year’s report, Ken Bogart, John Lee, and I (all of us supposed veterans in the teaching of induction) were surprised by things I heard as students progressed through the chapter on mathematical induction which had been moved to the body of the notes. One of the first things I noted was that many students thought the Principle of Mathematical Induction is used exclusively for verifying formulas similar to $\sum_{i=1}^{n} i = n(n+1)/2$. While many of the usual first induction problems encountered are of this form, most of problems in these notes are more varied. Especially once they get to graph theory (but even for recurrences!), students were often at a loss to identify the statement to be proved, no less to identify and use the inductive process involved. Both Ken and I (and now John Lee) were intrigued by this from the point of view of understanding student learning, as well as from the view of the more practical problem of how to address this in the notes. In the second adaptation induction was moved much earlier and more problems were added, including a problem sequence which specifically addresses student understanding of inductive processes. In addition, about two-thirds of the way through the chapter we include a complete proof of a problem they’ve already been asked to work. As well as serving as a template for induction, the exposition includes comments on some common pitfalls. John thought these changes were well-received by his Fall 2005 class, but he also found the groups were still having trouble identifying the sequence of statements to be proved. We have fine-tuned the presentation so that part of the procedure has been emphasized.

Another significant mathematical difference between the original notes and the adaptations is an emphasis on considering counting from the perspective of equivalence relations rather than relying on the Quotient Principle. In addition to having wider application to the case of equivalence classes of unequal size, using equivalence
relations is an important skill to be obtained from a beginning course in discrete mathematics and a precursor to cosets and other important partitions in abstract algebra. The current edition has a full chapter on equivalence relations, absorbing much of the material from the original appendix on equivalence relations as well as problem sequences from Ken’s first chapter which used the Quotient Principle. In addition, the material on distributions has been reworked in the context of equivalence relations.

2.1 A tabular list of topics in the current version

Below is a list of topics according to Ken’s sections giving the corresponding sections in the current notes. The sections which are now optional are given in *italics*.

<table>
<thead>
<tr>
<th>Topic</th>
<th>Ken’s</th>
<th>Adapt III (2006)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sum Principle</td>
<td>1.2.1</td>
<td>1.2</td>
</tr>
<tr>
<td>Product Principle</td>
<td>1.2.1</td>
<td>1.2</td>
</tr>
<tr>
<td>Functions</td>
<td>A.1;1.2.2</td>
<td>A; 1.2, 1.3, 2.3.1</td>
</tr>
<tr>
<td>Directed graphs</td>
<td>A.1;1.2.2</td>
<td>A; 1.3</td>
</tr>
<tr>
<td>Bijection Principle</td>
<td>1.2.3</td>
<td>1.3</td>
</tr>
<tr>
<td>Counting subsets</td>
<td>1.2.4</td>
<td>3.3</td>
</tr>
<tr>
<td>Pascal’s Triangle</td>
<td>1.2.5</td>
<td>3.3.1</td>
</tr>
<tr>
<td>Quotient Principle</td>
<td>1.2.6</td>
<td>(de-emphasized)</td>
</tr>
<tr>
<td>Lattice paths</td>
<td>1.3.1</td>
<td>3.3.2</td>
</tr>
<tr>
<td>Catalan numbers</td>
<td>1.3.1; 4.3.5</td>
<td>3.3.2</td>
</tr>
<tr>
<td>Binomial Theorem</td>
<td>1.3.2; 2.1.2</td>
<td>5.1</td>
</tr>
<tr>
<td>Pigeonhole Principle</td>
<td>1.3.3</td>
<td>1.4</td>
</tr>
<tr>
<td>Ramsey numbers</td>
<td>1.3.4; 2.1.5</td>
<td>1.5,3.5,4.5</td>
</tr>
<tr>
<td>Mathematical Induction</td>
<td>B.1-B.2 ; 2.1.1</td>
<td>B; Chapter 2</td>
</tr>
<tr>
<td>Binomial coefficients</td>
<td>2.1.2</td>
<td>3.3</td>
</tr>
<tr>
<td>Inductive definition</td>
<td>2.1.3</td>
<td>Chapter 2</td>
</tr>
<tr>
<td>General Product Princ</td>
<td>2.1.4</td>
<td>2.3</td>
</tr>
<tr>
<td>Double Induction</td>
<td>2.1.5</td>
<td>3.5</td>
</tr>
<tr>
<td>Asymptotic combinatorics</td>
<td>2.1.6</td>
<td>4.6</td>
</tr>
<tr>
<td>Equivalence relations</td>
<td>A.2</td>
<td>Chapter 3</td>
</tr>
<tr>
<td>Recurrences</td>
<td>2.2; 4.3</td>
<td>2.2.1;5.3</td>
</tr>
</tbody>
</table>
### Topic | Ken’s | Adapt III (2006)
---|---|---
Undirected graphs | 2.3.1 | 4.1
Walks and paths | 2.3.2 | 4.2
Labelled trees | 2.3.3 | 4.3
Spanning trees | 2.3.4 | 4.4
Minimal spanning trees | 2.3.5 | 4.4
Deletion/contraction | 2.3.6 5.3 | 4.4.1; 6.5.1
Shortest paths | 2.3.7 | 4.5

| Topic | Ken’s | Adapt III (2006) |
---|---|---|
Ordered-functions | 3.1.2 | 3.4 |
Multisets | 3.1.3; 5.2.1 | 3.4 |
Compositions of integers | 3.1.4 | 2.2; 3.3 |
Broken permutations | 3.1.5 | 7.1 |
Stirling numbers | 3.2 | — |
Partitions of integers | 3.3 | — |

| Topic | Ken’s | Adapt III (2006) |
---|---|---|
Picture functions | 4.1.2 | 5.1 |
Generating polynomials | — | 5.2.1 |
Generating functions | 4.1.3 | 5.2.2 |
Formal power Series | 4.1.4 | 5.2.2 |
Product Principle for gf | 4.1.5 | 5.2.3 |
Extended Binomial Thm | 4.1.6 | — |
Gf for integer partitions | 4.2 | — |
Gf and recurrences | 4.3 | 5.3 |
Partial fractions | 4.3.4 | — |

| Topic | Ken’s | Adapt III (2006) |
---|---|---|
Size of unions | 5.1.1-5.1.2 | 1.2; 6.1 |
Inclusion and Exclusion | 5.1.3 | 6.2 |
Ménage Problem | 5.2.2 | 6.4 |
Counting onto functions | 5.2.3 | 6.3 |
Chromatic polynomial | 5.2.4 | 6.5 |

| Topic | Ken’s | Adapt III (2006) |
---|---|---|
Groups acting on sets | 6.1-6.3 | — |

#### 2.2 The Instructor Handbook

An important feature of my grant proposal was the construction of an instructor handbook to accompany the notes. This has taken form over the past year, and it
has become a joint project with John Lee. In general terms, we envision an instructor supplement which will be motivational as well as pass on practical advice that we’ve learned while teaching the course. At this point, the booklet includes an overview of the Bogart method; an overview on the changes in the adaptation with some discussion of reasons for it; anecdotal information and advice from the OSU instructors. Separate chapter summaries which can be modified by each instructor have also been included. In this year’s evaluation report, Barbara Edwards has suggested additional topics to be considered.

3 The Adapted Method—An overview

Up to now, in this report I’ve tried to separate the content from the method. In this section is an overview of the method as implemented in Fall 2004 and Fall 2005. Because this is an overview, some of this material appeared in last year’s report. Many of the points raised in this section will be discussed in the instructor handbook.

During the project, the course has been offered as a 4-credit course and the classes have meet two afternoons a week in 100-minute blocks. Both John and I found offering the course in two long blocks to be an advantage since it did a good job of modeling how we expected students to work outside the classroom. Also, two meeting times a week uses less “startup” time than offering the course three or four days a week. To assist a re-alignment of requirements for our majors, we’ve suggested that the course be changed to a 3-credit course meeting in two 75-minute blocks.

Although student time in class will be reduced, the expectation for the amount of work outside the classroom would be about the same. The usual format for the classes was to assign a range of problem numbers on Monday—a minimal assignment to be completed by the beginning of class on the following Monday. Occasionally each of us modified the list at the end of the week, but in general we saw the wisdom in setting expectations which the students could strive to meet. Although for the beginning material it was reasonable to expect the students to work about 20 problems a week, later problems are harder and often their solution requires making connections with earlier information. On average, 12-15 problems per week was the usual expectation and this required a healthy amount of time outside the classroom. This question of benchmarks (and whether they’re important) is another topic for the instructor handbook.

Each of us used whole-class discussion time to help break up the time. I used this type of time sparingly during the term, usually at the 85-minute mark when students seemed especially tired or frustrated. On the other hand, John used whole-class time fairly regularly at the 60-minute mark, and this year (Fall 2006) the time was often spent having groups collectively explain a problem or some features of a problem to the whole class. Barbara gives some information on student views of the necessity of whole-class time in this year’s report. The amount of whole-class participation and the various forms it might take is an important topic for our instructor handbook.

John and I agree that quick and frequent feedback is essential, and we each collected written work once a week and provided a 2-day turnaround on graded work. We
recommend this frequent feedback and quick turnaround, but recognize this is a feature which might change as others teach the course. Ken’s method allowed for unlimited re-submission of problems, but we restricted both the number of resubmissions and the time window for resubmission. We also changed Ken’s 0-5-9-10 “triage” grading-scale slightly by adding a possible grade of 7, partly as a compensation for fewer re-submittals. What problems were graded? In the first part of the Fall 2004 term, my students were expected to turn in all assigned problems, and I graded about 5 problems per week. As the term progressed and my expectation of the quality of their write-ups increased, my procedure evolved to selecting a subset of the assigned problems which would be graded. On the other hand, at the end of every week John specified which problems he would grade on the following Tuesday, and he diligently assigned two grades for each problem he graded, with one grade specifically assessing the mathematical exposition.

One last feature of the method is: How our students motivated to do more? In the Fall 2004 I used a special course-grade protocol to entice one group to do more problems. As the term evolved, each week these students and I would come to an agreement on what they would hand, and the stronger two students substituted a final problem set for the final. In John’s class, every group proceeded at the same rate. As I said earlier, previous supplemental sections now appear as optional sections in the main body of the book. It’s hoped that this move will encourage more students to look at more material.

4 What effect has this course had on our majors?

No provision was made for any longitudinal analysis of the longterm effect of this course and guided discovery on our majors. Barbara Edwards’ report does include an assessment of student attitudes six months after taking the Fall 2005 course.

John Lee frequently teaches in the advanced calculus sequence which math majors are encouraged to begin the same term as they take this course. John is especially impressed by how the choice of topics in this course as well as the guided discovery method helps to foster and develop important mathematical instincts and abilities. In addition, we agree that our discussions have changed the way each of us teaches other courses, and that experimentation in turn informs the continuing adaptation.

John and I have been involved in a local discussion of what we expect of our majors. Some colleagues identify an improvement in proof-writing skills as being the ultimate test of our success with majors. For me, this goal is too narrow. (And I also question whether proof skills can be properly assessed, since improvement can be glacial and also it comes in spurts which can’t be predicted.) Rather, for me the objective is a discernible development of the students’ overall mathematical sophistication. Looking at guided discovery from this point of view, it approaches mathematics very much like a mathematician does when on unfamiliar ground. As we indicate in the current preface: Students are asked to work on how to think about mathematics by looking at special cases, by trying to discover patterns, by wandering up blind alleys, by possibly being frustrated, and finally by putting it all together into a solution of a problem or
proof of a theorem. This thinking process is as important to a student of mathematics as the specific discrete mathematics learns. It is also hoped that the students also see that putting it all together sometimes includes solving the same problems again from different points of view, with each point of view shedding new light on the result.

5 Concluding Thoughts

As I said earlier, work on the notes has taken more time than I had anticipated—in fact, the work on the notes alone long ago surpassed the amount of time I had originally allocated for the whole project. I expect Dr. Orellana’s comments will result in still more changes, changes which I will happily make as basic to ensuring the spirit of guided discovery is maintained.

Work on the project has also resulted in more time spent in discussions with John Lee (and other colleagues to a lesser extent), discussions which are invaluable and have influenced my general teaching. The high level of these discussions was also unanticipated and is a priceless bonus.

In this last year of the project, more of my time must be devoted to dissemination—both in the form of talks and also writing up our observations for publication. Because this kind of presentation is different for me, it will require a lot of time but I expect to find it satisfying. To do it well will probably require my concentrating on venues which would seem to have the most impact. I plan to continue disseminating information about the course and OSU’s implementation of the method after the grant is finished.