Course Preview

- Mth 599/636/376 meets MW 10:00
- Students needing third credit (636/676) will give presentation on an agreed topic for third credit.
- Finish Applications of Duality
- ARs, ANRs, AEs, ANEs
- Čech-Alexander-Spanier Cohomology
- Proof of Alexander and Lefschetz Duality
- Künneth Theorem
- Other topics

Other Applications of Duality

- If $M^n$ is orientable w/ boundary, then $\partial M$ is orientable.
- If $K$ is a compact, locally contractible proper subset of $S^n$, then $\tilde{H}_i(S^n - K) \simeq H^{n-i-1}(K)$
- If $M$ is a compact 3-manifold with boundary, and $H_1(M) \cong 0$, then $\partial M$ is a union of 2-spheres
- If $M$ is a compact manifold with boundary, $\partial M$ is not a retract of $M$
- If $M^n$ is compact contractible with boundary, then $\partial M$ and $2M$ are homology spheres
- A nonorientable $n - 1$ closed manifold cannot be embedded in $S^n$ or $R^n$

ANRs, ANEs

- $X$ is an ANR($C$) (AR($C$)) if whenever $X$ is embedded via $e$ in a $C$ space $Y$ as a closed subspace, then $e(X)$ is a retract of some nbhd of $e(X)$ in $Y$ ($e(X)$ is a retract of $Y$).
- $X$ is an ANE($C$) (AE($C$)) if whenever $A$ is closed in a $C$ space $Y$ and $f : A \rightarrow X$ is a map, there exists an extension of $f$ to a map $F$ defined on a neighborhood of $A$ in $Y$ ($F$ defined on all of $Y$).

Relationship between ANEs and ANRs

**Theorem:** Suppose that $C$ is preserved by forming adjunction spaces. Then :

- $X$ is an ANE($C$) if and only if $X$ is an ANR($C$)
- $X$ is an AE($C$) if and only if $X$ is an AR($C$)

**Examples**

- $R^n$, $I^n$, $I^0$ are ARs for the normal spaces and for separable metric spaces.
- $S^n$ is an ANR for normal spaces and separable metric spaces.