**Major Theorems Last Term:**

**Embedding Theorem:** Every separable metric space is homeomorphic to a subspace of $\beta I \equiv [0, 1]^{\omega}$.

**Tychonoff’s Theorem:** A product of compact spaces is compact.

**Urysohn’s Lemma:** If $A, B$ are closed, disjoint in $X$, there exists a map $f : X \to [0, 1]$ such that $f(A) = \{0\}$ and $f(B) = \{1\}$.

**Tietze’s Extension Theorem:** If $A \subset X$ is closed and $f : A \to Y$ is continuous where $Y$ is either $\mathbb{R}$ or an interval in $\mathbb{R}$, then there exists a continuous function $F : X \to Y$ extending $f$.

**Baire’s Theorem:** If $X$ is either compact Hausdorff or complete metric, then a countable intersection of dense and open subspaces of $X$ is dense.

**Other Major Theorems:**

**Cantor Image Theorem:** Every separable metric space is the continuous image of a subspace of the Cantor set.

**Urysohn Metrization Theorem:** Every regular space with a countable basis is metrizable.

**Borsuk’s Homotopy Extension Theorem:** Suppose $f : X \to U$ is a map into an open subset $U$ of $\mathbb{R}^{n}$, $A$ is closed in $X$ and $G : A \times [0, 1] \to U$ is a map with $G(a, 0) = f(a)$. If $X \times I$ is normal, then there is a map $F : X \times [0, 1] \to U$ so that $F(x, 0) = f(x)$ and $F(a, t) = G(a, t)$ for each $a \in A$.

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**Preview of topics in Mth 532**

* Completeness (§ 43, 48)
* Topologies on spaces of functions (§ 46)
* Metrization Theorems (§ 39-42)
* More on Compactifications (§ 38)
* Fundamental Group and Covering Spaces (Ch. 9, 11, 13)
* Classification of Surfaces (Ch. 12)

**Assignment:** read section 43 on completeness and start on the homework.

Also, review the definition and theorem on homotopy on the next page.

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**Def.** Maps $f$ and $g$ from $X$ to $Y$ are **homotopic** if there exists a map $H : X \times [0, 1] \to Y$ such that:

$$H(x, 0) = f(x) \quad \text{and} \quad H(x, 1) = g(x)$$

The map $H$ is called a **homotopy** from $f$ to $g$.

**Theorem:**

Homotopy is an equivalence relation on maps from $X$ to $Y$.

**Proof:** Last term.

**Example:** Any two maps $f$ and $g$ into a convex subset of $\mathbb{R}^{n}$ are homotopic via $H(x, t) = t \cdot f(x) + (1 - t) \cdot g(x)$. 