Covering Maps

Read Section 53.

**Def:** Let \( p : E \rightarrow B \) be a surjective map. An open set \( U \) in \( B \) is *evenly covered* by \( p \) if \( p^{-1}(U) \) can be written as a disjoint union \( \bigcup V_\alpha^{\text{open}} \) such that for each \( \alpha \), \( p|_{V_\alpha} : V_\alpha \rightarrow U \) is a homeomorphism.

**Def:** Let \( p : E \rightarrow B \) be a surjective map. If every point \( b \in B \) has a neighborhood \( U \) evenly covered by \( p \), \( p \) is called a *covering map* and \( E \) is said to be a *covering space* of \( B \).

**Examples:**

Relation to Local Homeomorphism

**Thm:** \( p : R \rightarrow S^1 \) given by

\[
p(t) = (\cos 2\pi t, \sin 2\pi t) = e^{2\pi it}
\]

is a covering map.

**Note:** Covering maps are local homeomorphisms, but the converse isn’t necessarily true.

**Examples:**

Relation to Subspaces

**Thm:** Let \( p : E \rightarrow B \) be a covering map. If \( C \) is a subspace of \( B \) and \( D = p^{-1}(C) \), then

\[ p|_D : D \rightarrow C \]

is a covering map.

**Thm:** If \( p : E \rightarrow B \) and \( q : F \rightarrow C \) are covering maps, so is

\[ p \times q : E \times F \rightarrow B \times C \]

**Examples:**