Chapter 1: Introduction, Sec 1.1, 1.2, 1.3

1. Definitions:
   (a) Differential Equation, Mathematical Model
   (b) Direction (Slope) Field, Equilibrium Solution
   (c) Rate (Growth) constant
   (d) Initial Condition, Initial Value Problem (IVP)
   (e) General Solution, Integral Curves
   (f) Ordinary Differential Equation (ODE), Partial Differential Equation (PDE)
   (g) Systems of Differential Equations
   (h) Order, Linear, Nonlinear, Linearization

2. Important Skills:
   (a) Derive differential equations that mathematically model simple problems. (Example 1, p. 2; Also see p. 7)
   (b) Know what an initial value problem is, and how to show a given function is a solution to one. (Example 2, p. 13)
   (c) Know the difference between an ordinary differential equation and partial differential equation. (p. 19)
   (d) Know how to classify differential equations using order and linearity. (pp. 19-21)

Chapter 2: First Order Differential Equations, Sec 2.1, 2.2, 2.3, 2.4, 2.5, 2.6

1. Definitions:
   (a) First Order Ordinary Differential Equation
   (b) Integrating Factor, Integral Curves
   (c) Separable differential equations
   (d) Homogeneous differential equations (p. 49)
   (e) Implicit solutions (p. 74)
   (f) Logistic equations, intrinsic growth rate (p. 79)
   (g) Existence and Uniqueness of Solutions, General Solutions
(h) Autonomous, Logistic Growth, Equilibrium Solutions
(i) Stable solutions, asymptotically stable solutions, unstable equilibrium solution (p. 83)
(j) Threshold (p. 85)
(k) Critical Points (p. 88)
(l) Integrating factors, Exact equations (pp. 94-98)

2. Theorems:

(a) Theorem 2.4.1: Existence and uniqueness of solutions to linear first order ODE’s.
(b) Theorem 2.4.2: Existence and uniqueness of solutions to first order IVP’s
(c) Theorem 2.6.1: Existence and uniqueness of solutions to exact first order ODE’s.

3. Important Skills:

(a) Be able to determine if a first order differential equation is linear or nonlinear. Equation (3) on p. 32 gives the form for a linear ODE.
(b) If the differential equation is linear, compute the integrating factor, and then the general solution (p. 36)
(c) If it’s nonlinear, is it separable? If it’s separable, you will need to compute two different integrals.
(d) If the differential equation is not separable, is it exact? If so, solve it using the method in Section 2.6 (Example 2, p. 97)
(e) If it isn’t separable or exact, try to find an integrating factor (of just one variable) that will make it exact.
(f) It is crucial to know integration of basic functions and integral methods from your calculus courses. For example, various substitutions, integration by parts, and partial fractions will all be utilized.
(g) What happens to solutions as time tends to infinity (end behavior)? Understand stability, asymptotic stability and instability.
(h) Understand the three steps in the process of mathematical modeling: construction of the model, analysis of the model, and comparison with experiment or observation. (Example 1 and p. 52)
(i) Understand Relevant Applications: Mixing Problems, Compound Interest, etc.
(j) Determine the existence and uniqueness of solutions to differential equations. (Examples on pp. 70-75)
(k) Know how to recognize autonomous equations, and utilize the direction field to represent solutions to them. Be able to determine asymptotically stable, semi-stable, and unstable equilibrium solutions. (Example 1, p. 83)
(l) Know the concepts of a threshold value and carrying capacity. (Section 2.5)
Chapter 3: Second Order Linear Equations, Sec 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.7, 3.8

1. Definitions:
   (a) Linear vs. nonlinear
   (b) Homogeneous vs. nonhomogeneous
   (c) Characteristic Equation
   (d) Principle of superposition (p. 145)
   (e) Linear Independence and dependence, Wronskian
   (f) General Solution, Fundamental Set of Solutions
   (g) Particular Solution
   (h) Method of Undetermined Coefficients
   (i) Method of Variation of Parameters
   (j) Period, Natural Frequency, Amplitude, Phase
   (k) Overdamped, Critically Damped, Underdamped
   (l) Resonance
   (m) Transient Solution, Steady-State Solution or Forced Response

2. Theorems:
   (a) Theorem 3.2.1: Existence and uniqueness of solutions to
      \[ y'' + p(t)y' + q(t)y = g(t); \quad y(t_0) = y_0, \quad y'(t_0) = y'_0 \]
   (b) Theorem 3.2.2: Principle of superposition. If \( y_1(t) \) and \( y_2(t) \) are solutions to (2), so is \( c_1y_1(t) + c_2y_2(t) \) for any constants \( c_1 \) and \( c_2 \).
   (c) Theorem 3.2.3: Finding solutions to initial value problems using the Wronskian at the initial conditions.
   (d) Theorem 3.2.4: Representing general solutions to second order linear homogeneous ODE’s
   (e) Theorem 3.2.6: Abel’s Theorem.
   (f) Theorem 3.5.1: Relating differences in nonhomogeneous solutions to fundamental solutions (used to prove the following theorem).
   (g) Theorem 3.5.2: General solutions to linear nonhomogeneous ODE’s.
   (h) Theorem 3.6.1: General solutions to linear nonhomogeneous ODE’s (using variation of parameters to determine the particular solution).

3. Important Skills:
(a) Be able to determine if a second order differential equation is linear or nonlinear, homogeneous, nonhomogeneous. (If it can be put into the form given by Equation (3) in p. 136, it is linear.)

(b) Can you recognize a homogeneous equation with constant coefficients, and derive the characteristic equation? (Example 3, p. 147) This equation will be quadratic, so know the quadratic formula, the types of roots one gets: real and distinct, repeated, or complex conjugate. These three cases will be crucial to the types of solutions one gets to constant coefficient linear differential equations.

(c) Be able to write down fundamental solution sets to homogeneous equations. This means finding two linearly independent solutions. You can use the Wronskian to show that two solutions are linearly independent. (Example 3, p. 147)

(d) What are the fundamental solution sets for each of the three cases of roots when solving constant coefficient equations? The summary is on p. 170. (Example 3, p. 147; Example 2, p. 162; Example 2, p. 169)

(e) Reduction of order is a way to take a known solution and produce a second linearly independent one. (Example 3, p. 171)

(f) Solutions to second order nonhomogeneous equations have two components. There is the homogeneous solution, or complementary solution, and the particular, or nonhomogeneous solution (Theorem 3.5.2, p. 175).

(g) To find particular solutions you must know the Method of Undetermined Coefficients (see Table 3.5.1, p. 181) and Variation of Parameters. (Example 4, p. 178; Example 1, p. 185)

Chapter 4: Higher Order Linear Equations, Sec 4.1, 4.2, 4.3, 4.4

1. Definitions:
   (a) n-th Order Linear ODE
   (b) Fundamental Set of Solutions, General Solution
   (c) Homogeneous and Nonhomogeneous Equations
   (d) Linear Dependence and Independence
   (e) Characteristic Polynomial, Characteristic Equation
   (f) Undetermined Coefficients
   (g) Variation of Parameters

2. Theorems:
   (a) Theorem 4.1.1: Existence and uniqueness of solutions to higher order linear ODE’s.
   (b) Theorem 4.1.2: General solutions to higher order linear ODE’s and the fundamental set of solutions
3. Important Skills:

(a) The methods for solving higher order linear differential equations are extremely similar to those in the last Chapter. There is simply n times the fun! The general solution to an n-th order homogeneous linear differential equation is obtained by linearly combining n linearly independent solutions. (Equation 5, p. 220)

(b) The generalization of the Wronskian is given on p. 221. It is used as in the last Chapter to show the linear independence of functions, and in particular homogeneous solutions.

(c) For the situation where there are constant coefficients, you should be able to derive the characteristic polynomial, and the characteristic equation, in this case each of n-th order. Depending upon the types of roots you get to this equation, you will have solution sets containing function similar to those in the second order case. (Examples 2-4, pp. 227-229)

(d) The general solution of the nonhomogeneous problem easily extends to the n-th order case. (Equation 9, p. 222)

(e) Both variation of parameters, and the method of undetermined coefficients generalize to determine particular solutions in the higher dimensional situation. (Example 3, p. 234; Example 1, p. 239)

Chapter 6: The Laplace Transformation, Sec 6.1, 6.2, 6.3, 6.4, 6.5, 6.6

1. Definitions:

(a) Laplace Transform, Kernel
(b) Improper Integral
(c) Piecewise Continuous function
(d) Unit Step Function (Heaviside Function)
(e) Unit Impulse Function, Delta Function
(f) Convolution

2. Theorems:

(a) Theorem 6.1.1: Comparison Test for Improper Integrals
(b) Theorem 6.1.2: Existence of the Laplace Transform, \( F(s) \)
(c) Theorem 6.2.1: Laplace Transform of \( f'(t) \)
(d) Corollary 6.2.2: Laplace Transform of \( f^{(n)}(t) \).
(e) Theorem 6.3.1: Transform of the unit step function, \( u_c(t) \), times a shifted function, \( f(t - c) \)
(f) Theorem 6.3.2: Inverse Transforming \( F(s - c) \)
(g) Equation (6.5.13): Laplace Transform of $\delta(t - t_0)$.

(h) Theorem 6.6.1: Convolution Result or Inverse Transforming $F(s)G(s)$

3. Important Skills:

(a) The Laplace transformation is defined through an improper integral. You must be comfortable evaluating them. Hence you should review this topic in any calculus book.

(b) Be able to calculate the transform of all the basic functions, given in the table 6.2.1 on p. 319. (Examples 5, 6 & 7, pp. 311-312) However, the table will be given to you in the exam. You should be able to use the table to calculate Laplace transforms and inverse Laplace transforms.

(c) Even more importantly, know how to compute inverse transform functions using manipulative translation methods. You may need to use partial fractions, but you should have already reviewed this for Chapter 2. (Examples 1 & 2, p. 320)

(d) Know how to transform derivatives of functions and linear differential equations. (Theorem 6.2.1 and Corollary 6.2.2, Examples 1 & 2, p. 320)

(e) Understand the unit step function, $u_c(t)$, as well as, the unit impulse function, $\delta(t)$, and how to use them in transforming and inverse transforming functions. (Example 2, p. 335; Example 1, p. 343). Also, solve initial value problems in which the forcing function is a step function or a translated (piecewise continuous) function.

(f) The process of using the Laplace transform method is as follows; Given a differential equation:

1) Transform both sides of the differential equation
2) Input the initial conditions (arising from transforming derivatives)

Note: Derivatives with respect to $t$ transform to polynomials in $s$. If the differential equation is linear, then the resulting equation is linear in $Y(s)$.

3) Solve the transformed equation for $Y(s)$
4) Invert the Laplace transform and recover $y(t)$ (using all the methods available to manipulate expressions into terms appearing in the table of transforms)

(Example 1, p. 320 for continuous forcing; Example 2, p. 335 for discontinuous forcing.)