4. Find the Laplace transform of the following convolution integral

\[ h(t) = \int_0^t (t - \tau)^2 \cos(2\tau) d\tau. \]

**Sol.** Theorem 6.6.1 states

\[ \mathcal{L} \left\{ \int_0^t f(t - \tau) g(\tau) d\tau \right\} = F(s)G(s). \]

In this case \( f(t) = t^2 \) and \( g(t) = \cos(2t) \) so that

\[ F(s) = \frac{2}{s^3} \]

and

\[ G(s) = \frac{s}{s^2 + 4}. \]

Thus, the Laplace transform of this convolution integral is

\[ H(s) = F(s)G(s) = \frac{2}{s^3} \frac{s}{s^2 + 4}. \]