1. Approximate each of the following integrals using the composite trapezoidal rule:

i. \[\int_0^2 e^{-x^2}dx\]

ii. \[\int_0^4 \frac{1}{1 + x^2}dx\]

iii. \[\int_0^{2\pi} \frac{1}{2 + \sin(x)}dx\]

iv. \[\int_0^1 \sqrt{x}dx\]

(a) For each integral, create a table of values \(T_n(f)\) for \(n = 2, 4, 8, \ldots, 512\). Also compute the difference between successive iterates \(T_{2n}(f) - T_n(f)\), and the ratio between successive differences \(\frac{T_{2n}(f) - T_n(f)}{T_{4n}(f) - T_{2n}(f)}\).

Your table should look something like:

<table>
<thead>
<tr>
<th>n</th>
<th>Approximation</th>
<th>Difference</th>
<th>Ratio</th>
</tr>
</thead>
</table>

Note: In order to see all significant figures, it is helpful to use something similar to the following when you output values in Matlab:

```matlab
a=0;
b=2;
n0=2;
f='exp(-x^2)';
[inT,diT,raT]=trapezoidal(a,b,n0,f);
fprintf('%d	%0.12f	%0.5e	%g
',n0*2^(i-1),inT(i),diT(i),raT(i))
```

(b) Comment if the trapezoidal rule performed worse or better than expected for each integral. Explain what may be the cause.
2. Repeat 1 using composite Simpson’s rule. Compare to trapezoidal with respect to accuracy and efficiency.

Note: In order to see all significant figures, it is helpful to use something similar to the following when you output values in Matlab:

```matlab
[inS,diS,raS]=simpson(a,b,n0,f);
fprintf('n \tIntegral \tError \t	Ratio
')
for i=1:length(inS),
    fprintf('%d	%0.12f	%0.5e	%g
',n0*2^(i-1),inS(i),diS(i),raS(i))
end
```

3. Regarding integral (i.), the asymptotic error formula for Simpson’s rule estimates that the number of subdivisions required to achieve an accuracy of $\epsilon = 10^{-10}$ is at least $n = 160$. For integral (ii.) $n = 396$ is required for an accuracy of $\epsilon = 10^{-12}$. Comment on whether your computational results agree or disagree with the asymptotic error formula. (See 5.2 Problem 6 in the text.)

4. Repeat 1 using Gaussian quadrature rule. Compare to trapezoidal and Simpson’s with respect to accuracy and efficiency.

Note: In order to see all significant figures, it is helpful to use something similar to the following when you output values in Matlab:

```matlab
[inG,diG,raG]=gausstable(a,b,n0,f);
fprintf('n \tIntegral \tError \t	Ratio
')
for i=1:length(inG),
    fprintf('%d	%0.12f	%0.5e	%g
',n0*2^(i-1),inG(i),diG(i),raG(i))
end
```

5. Optional: Improve $T_{512}(f)$ for each integral above by using the corrected trapezoidal rule (if it applies). Compare this approximation ($CT_{512}(f)$) to the Simpson’s rule approximation ($S_{512}(f)$) with respect to accuracy and efficiency.

6. Optional: Improve $T_{512}(f)$ for each integral above by using the Richardson’s extrapolation formula. Compare this approximation ($R_{512}(f)$) to the Simpson’s rule approximation ($S_{512}(f)$) with respect to accuracy and efficiency.