1. **451**: (10 points) Consider the linear system $Ax = b$ with

$$
A = \begin{bmatrix}
4 & 2 \\
-1 & 2 \\
\end{bmatrix}, \quad b = \begin{bmatrix}
10 \\
0 \\
\end{bmatrix}
$$

Assume that the right hand side is changed by adding a vector $\delta b$ with $|\delta b_1| \leq 0.1$ and $|\delta b_2| \leq 0.2$. This leads to a new system

$$
A(x + \delta x) = b + \delta b
$$

Show that

$$
\max_{i=1,2}|\delta x_i| \leq 0.12
$$

2. **551**: (10 points) Consider the perturbed linear system $(A + \delta A)(x + \delta x) = b + \delta b$. Prove that for $\|\delta A\|$ sufficiently small, then

$$
\frac{\|\delta x\|}{\|x\|} \leq \frac{\kappa(A)}{1 - \kappa(A)} \left( \frac{\|\delta A\|}{\|A\|} + \frac{\|\delta b\|}{\|b\|} \right).
$$

**Hint:** You will need the Banach Lemma: for $M$ square if $\|M\| < 1$, then $I - M$ is invertible and

$$
\|(I - M)^{-1}\| \leq \frac{1}{(1 - \|M\|)}.
$$

3. Look at [TB] 14.2

4. Look at [TB] 15.2 a

5. (10 points) Do [TB] 17.2

6. (10 points) Do [TB] 18.4