Toward Electromagnetic Characterization of Damage in Complex Dielectrics

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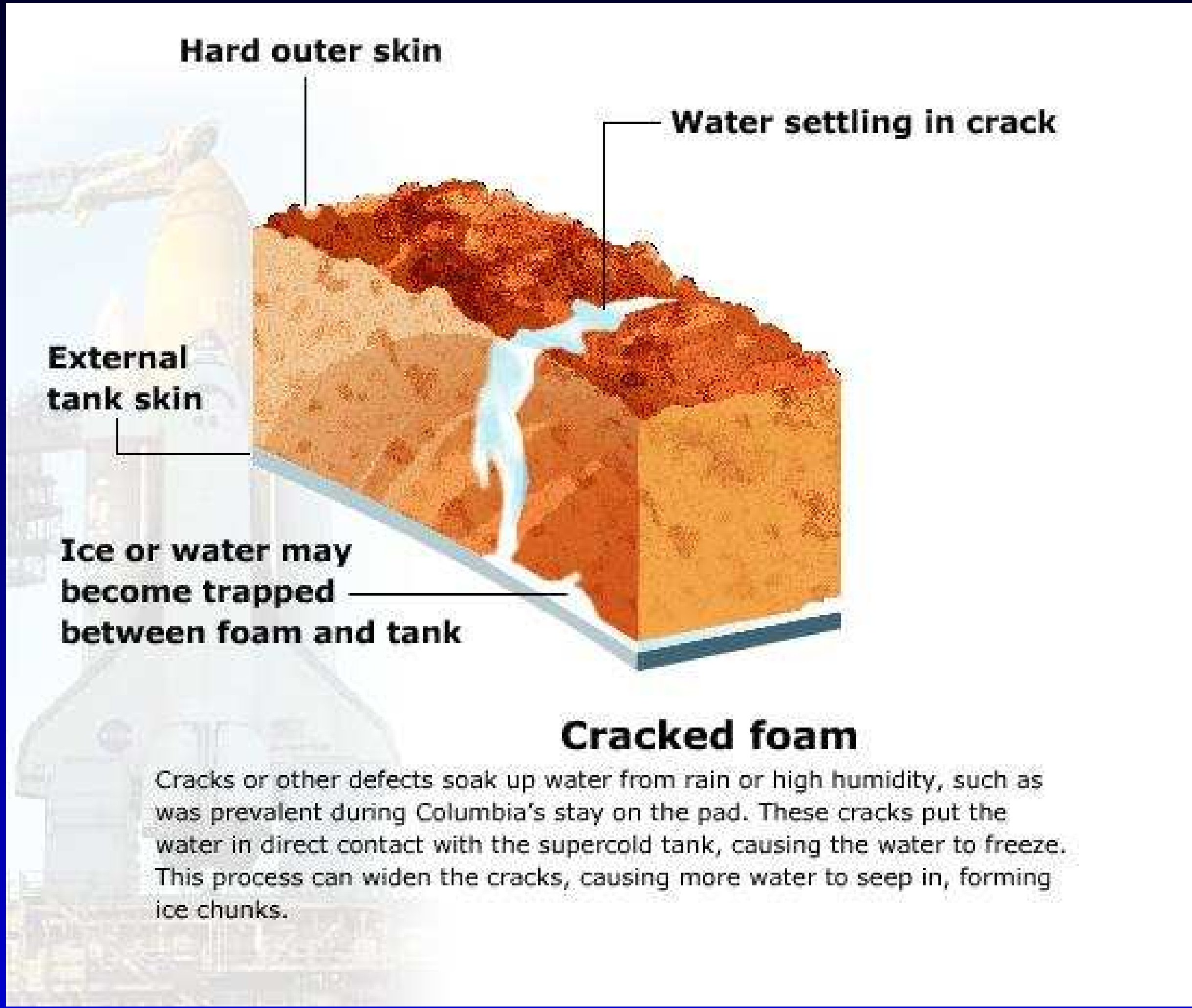
Motivating Application



The particular motivation for this research is the detection of defects in the insulating foam on the space shuttle fuel tanks in order to help eliminate the separation of foam during shuttle ascent.

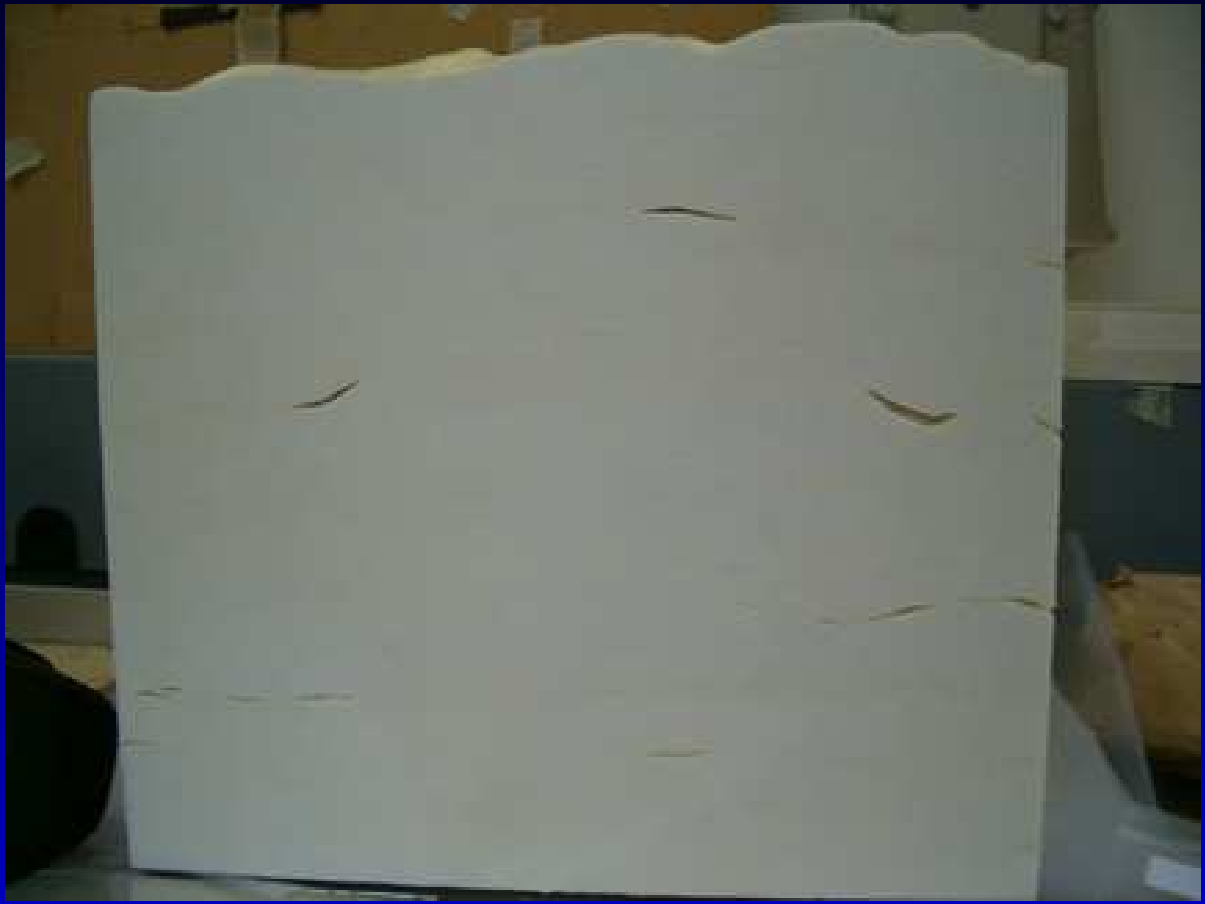
Our Contributions

- Gap Detection Inverse Problem
- Modeling Hetereogeneous Materials
  - Distributions of Dielectric Properties
  - Homogenization
  - Microstructure Modeling
- **2D Aspects**
  - Knit Lines
  - **Oblique Angles**
  - Focusing
  - **Obstacles**

Outline

- Damage Detection Problem for SOFI
- Difficulties
- Simplifications
- Some Computational Results
- Continuing Directions

Voids in Foam

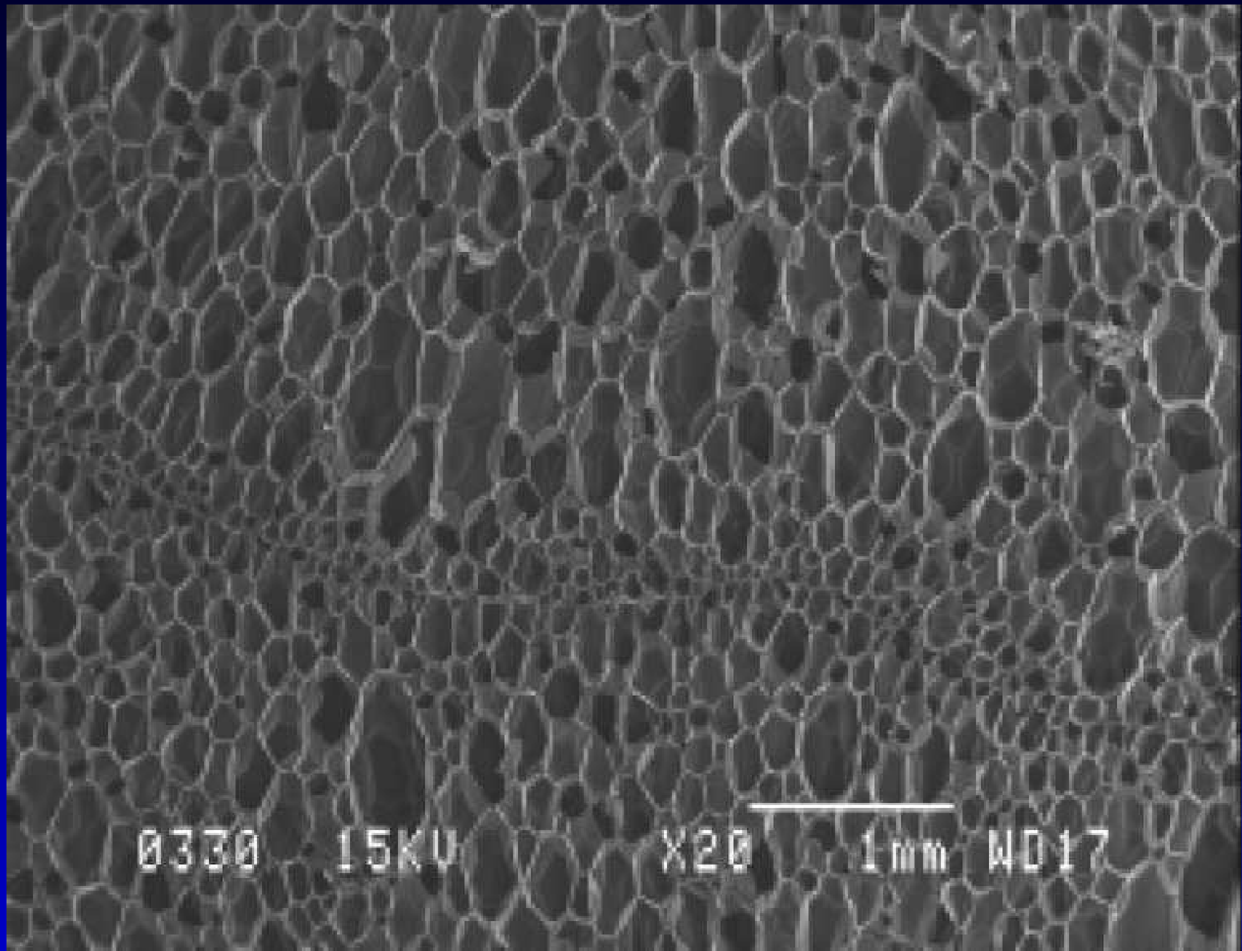


The foam on the space shuttle is sprayed on in layers (thus the acronym SOFI). Voids occur between layers.

Cured Layer

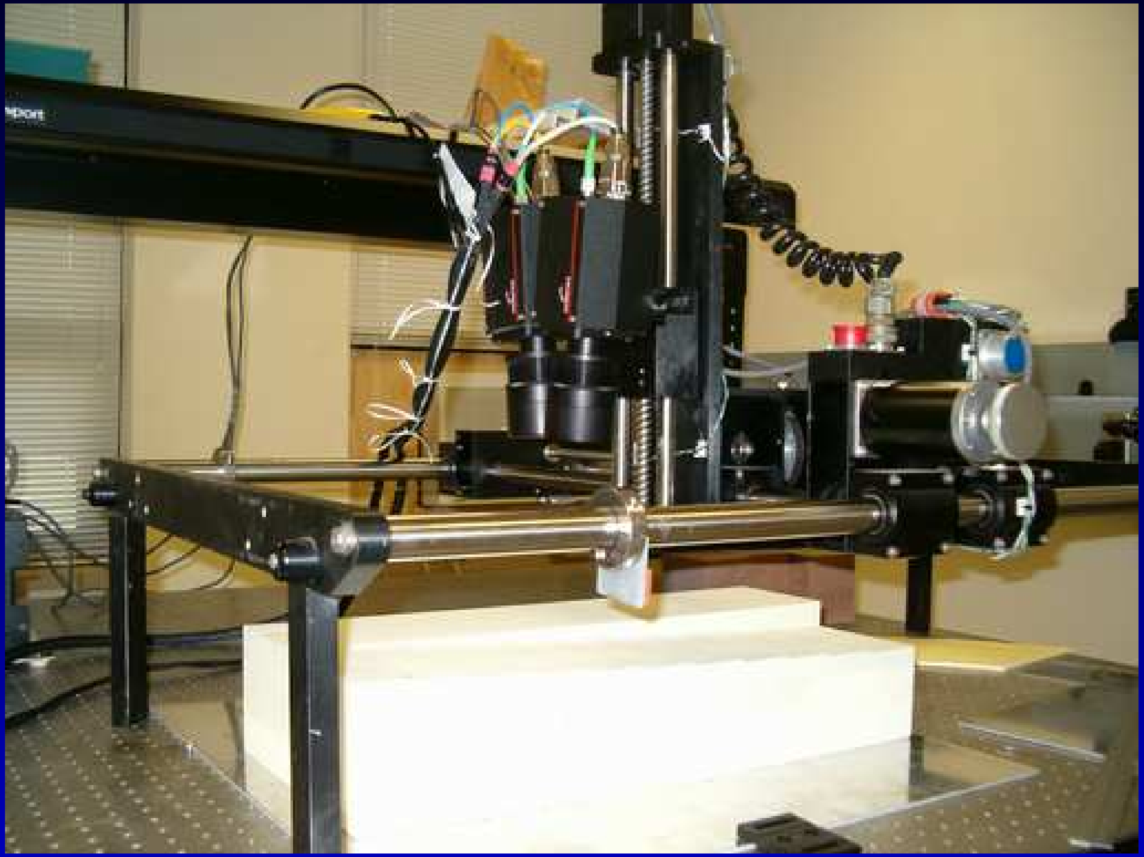
As the top of each layer cures, a thin knit line is formed which is of higher density (i.e., is comprised of smaller, more tightly packed polyurethane cells).

SOFI Under 20X Magnification



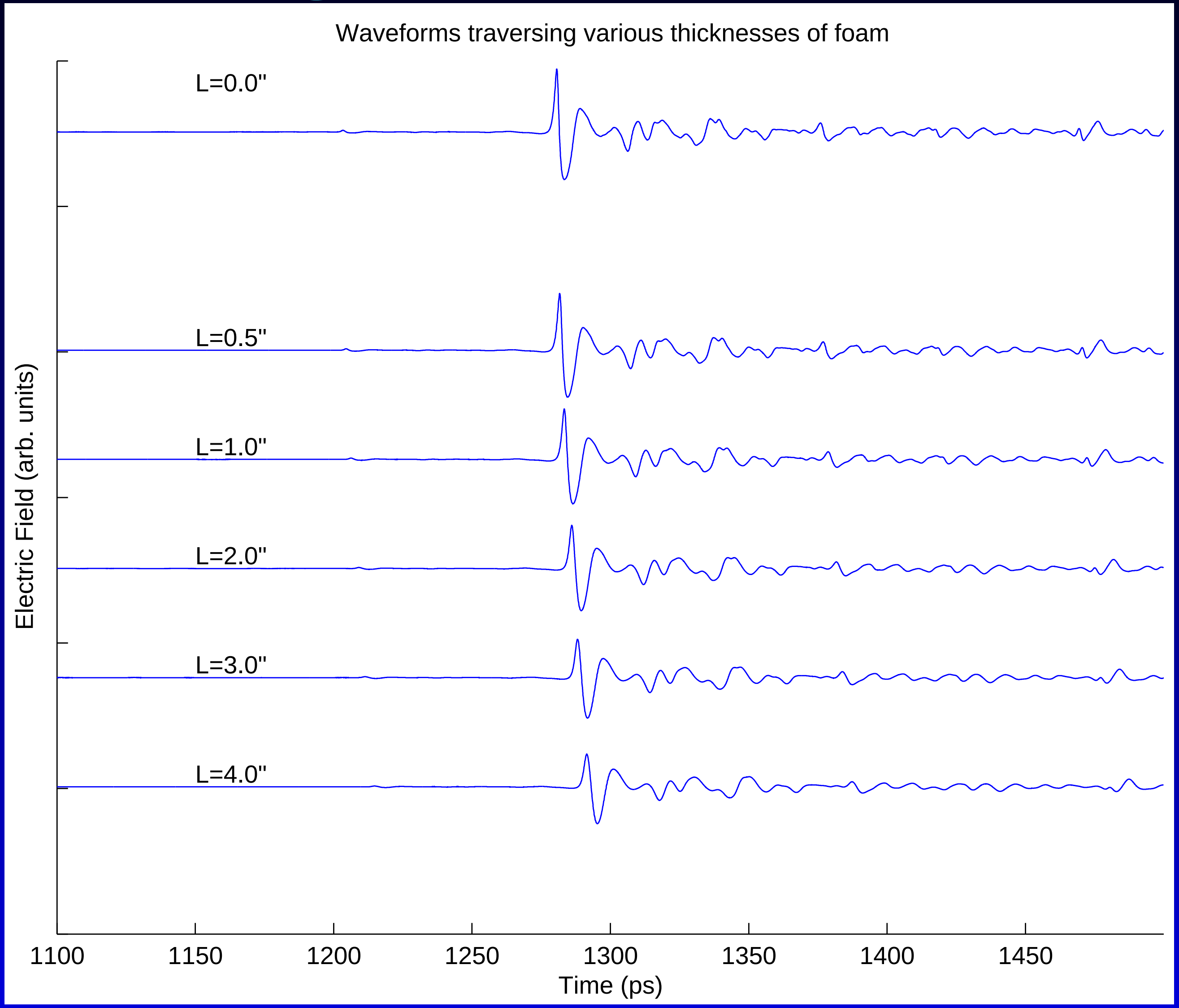
- Wavelength is on the order of 1mm; microstructure is smaller.
- Most loss in the material is due to scattering from faces.
- Numerical representation of cellular structure in an inverse problem is infeasible.

Picometrix T-Ray Setup



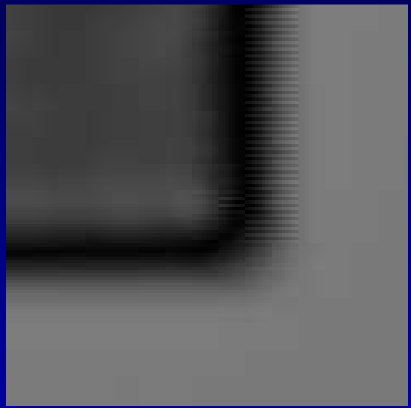
- Step-block can be turned upside down to sample varying gap sizes.
- Receiver and transmitter can be repositioned at various angles.
- Signal can be focused or collimated.

THz Through Foam



THz signal recorded after passing through foam of varying thickness, in a pitch-echo experiment.

Time-of-flight



Bitmap of time-of-flight recordings from step-block foam. Method clearly shows steep boundaries between foam and voids.

- Shows contrast, but does not accurately characterize damage.
- Less effective on horizontal discontinuities.

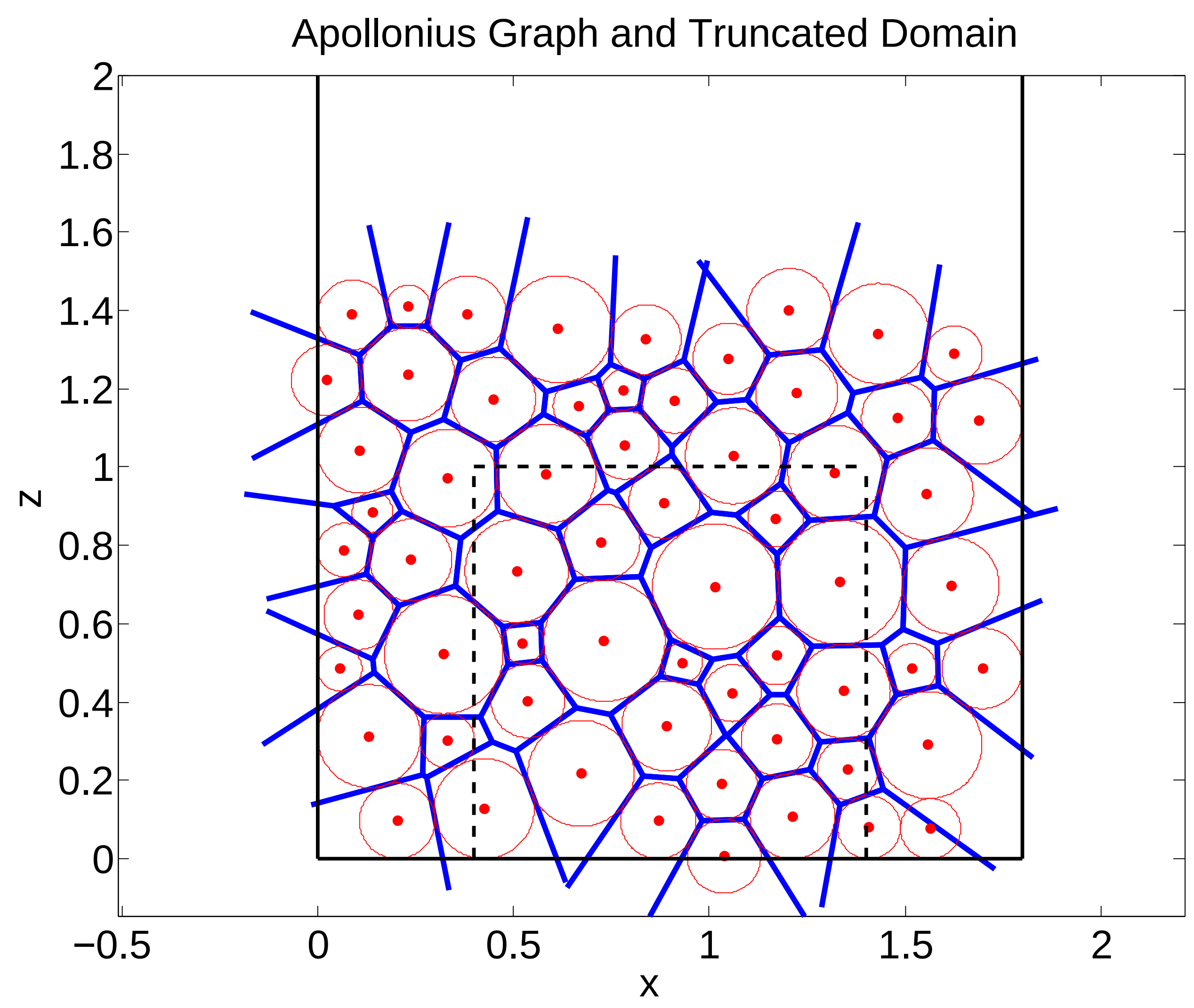
Difficulties

- Microstructure smaller than wavelength
- Domain is very large compared to wavelength
- Voids are only a few wavelengths large
- Index of material is close to air
- Ringing masks reflection information
- Scattering effects not well understood
- Obstacles

Microstructure

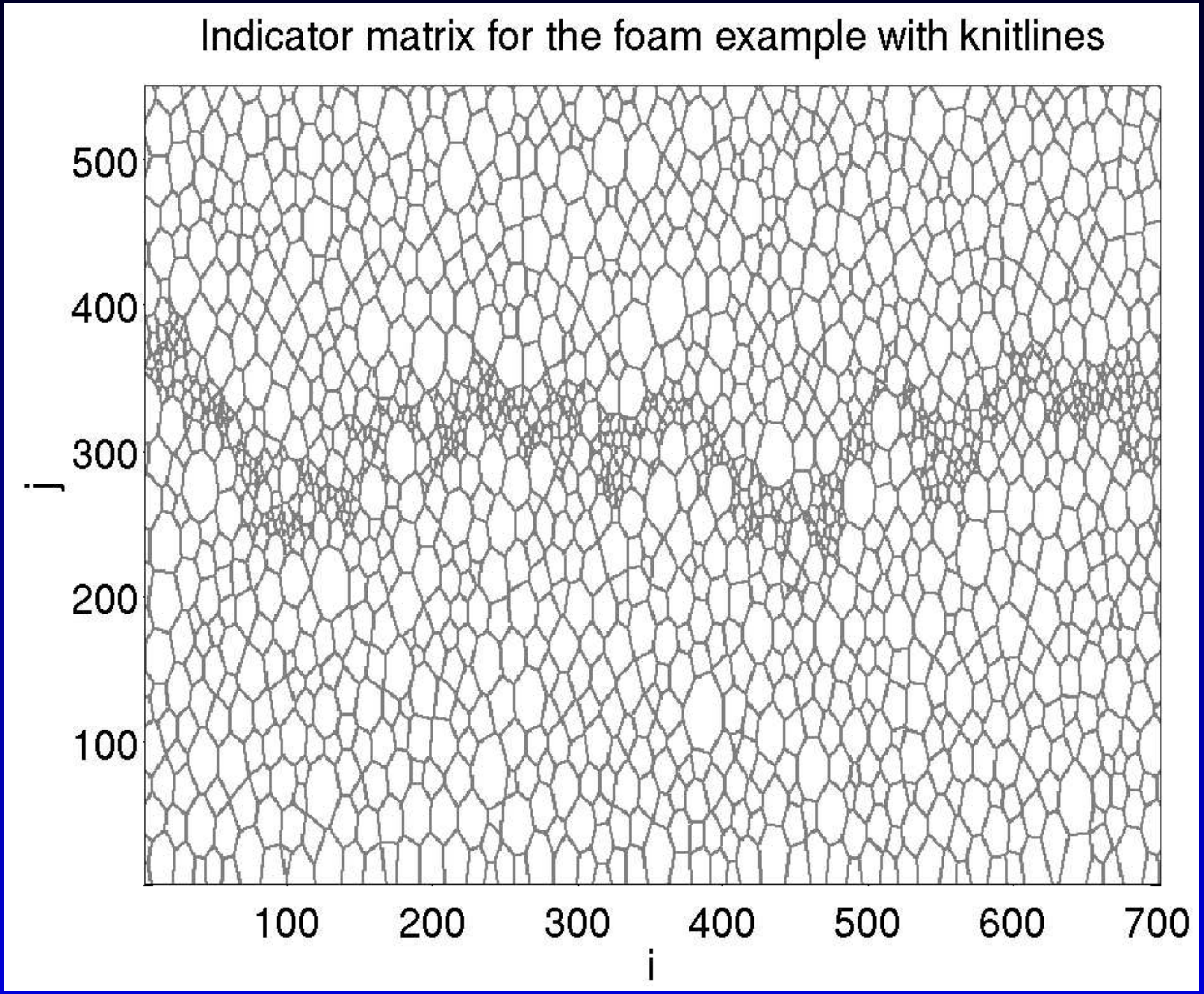
- Homogenization (Periodic Unfolding Method)
  - In collaboration with D. Cioranescu, et al.
  - Can give an effective permittivity based on microstructure
- Model random microstructures
  - Based on Appollonius tesselation of “random raindrops”
  - Distributions of statistical parameters yields heterogeniety
  - Can be used with constant wave speed.

Apollonius Graph



Apollonius graph on a close packing of disks generated by a “random raindrop” algorithm. The knit line is modelled using smaller drops.

Model of SOFI Microstructure



Apollonius graph is truncated and stretched, then edges are given a thickness and discretized to result in an indicator matrix as shown in the bitmap.

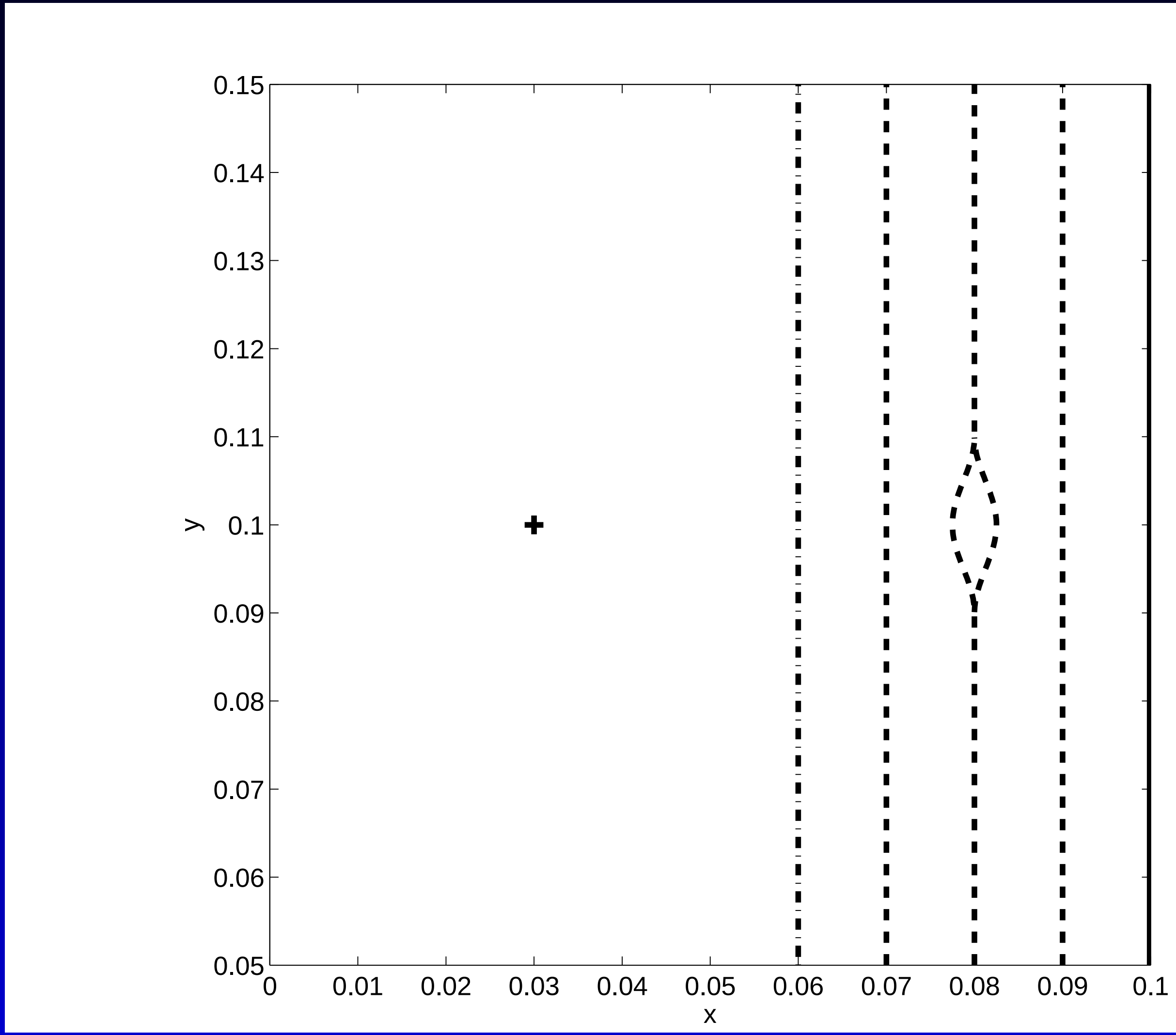
Simplifications

- Assume single-cycle pulse of fixed frequency
  - Possibly only tracking peak frequency
  - Possibly solving broadband problem in parallel
  - Maxwell's equations reduce to wave equation
- Assume homogenized material
  - For low frequency, microstructure is negligible
  - For fixed frequency, single wave speed
  - Possibly from homogenization method
- Assume 2D (uniformity in third)

2D Problem Outline

- Model
  - Equations
  - Boundary Conditions
- Computational Methods
- Sample Forward Simulations
- Inverse Problems
  - Without Obstacle
  - With Obstacle

Sample Domain with Void



Dashed lines represent knit lines, dot-dash is foam/air interface. Elliptical pocket (5 mm) between knit lines is a void. “+” marks the signal receiver. Back wall is perfect conductor.

2D Wave Equation

We assume the electric field to be polarized in the z direction, thus for $\vec{E} = (0, 0, E)$ and $\vec{x} = (x, y)$

$$\epsilon(\vec{x})\frac{\partial^2 E}{\partial t^2}(t, \vec{x}) - \nabla \cdot \left(\frac{1}{\mu(\vec{x})}\nabla E(t, \vec{x})\right) = -\frac{\partial J_s}{\partial t}(t, \vec{x})$$

where $\epsilon(\vec{x})$ and $\mu(\vec{x}) = \mu_0$ are the dielectric permittivity and permeability, respectively.

$$J_s(t, \vec{x}) = \delta(x)e^{-((t-t_0)/t_0)^4},$$

where $t_0 = t_f/4$ when t_f is the period of the interrogating pulse.

Boundary Conditions

Consider $\Omega = [0, 0.1] \times [0, 0.2]$

- Reflecting (Dirichlet) boundary conditions (right)

$$[E]_{x=0.1} = 0$$

- First order absorbing boundary conditions (left)

$$\frac{\partial E}{\partial t} - \sqrt{\frac{1}{\epsilon(\vec{x})\mu_0}} \frac{\partial E}{\partial x}\bigg|_{x=0} = 0$$

- Symmetric boundary conditions (top and bottom)

$$\left[\frac{\partial E}{\partial y}\right]_{y=0, y=0.2} = 0$$

We use homogeneous initial conditions $E(0, \vec{x}) = 0$.

Modeling Knit Lines/Void

- The speed of propagation in the domain is

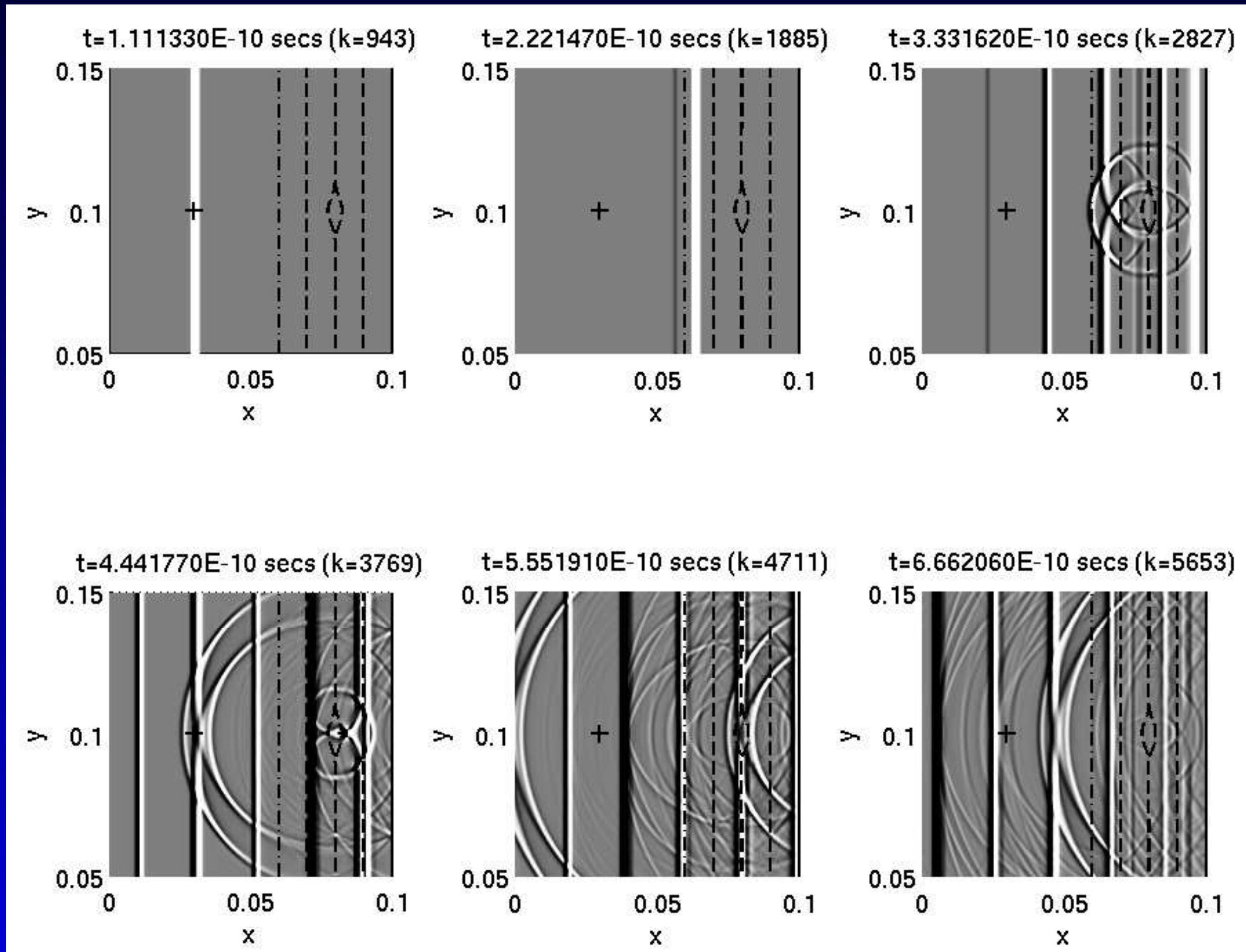
$$c(\vec{x}) = \frac{c_0}{n(\vec{x})} = \sqrt{\frac{1}{\epsilon(\vec{x})\mu_0}},$$

  where c_0 is the speed in a vacuum and n is the index of refraction.
- We may model knit lines or a void by changing the index of refraction, thus effectively the speed in that region.

2D Numerical Discretization

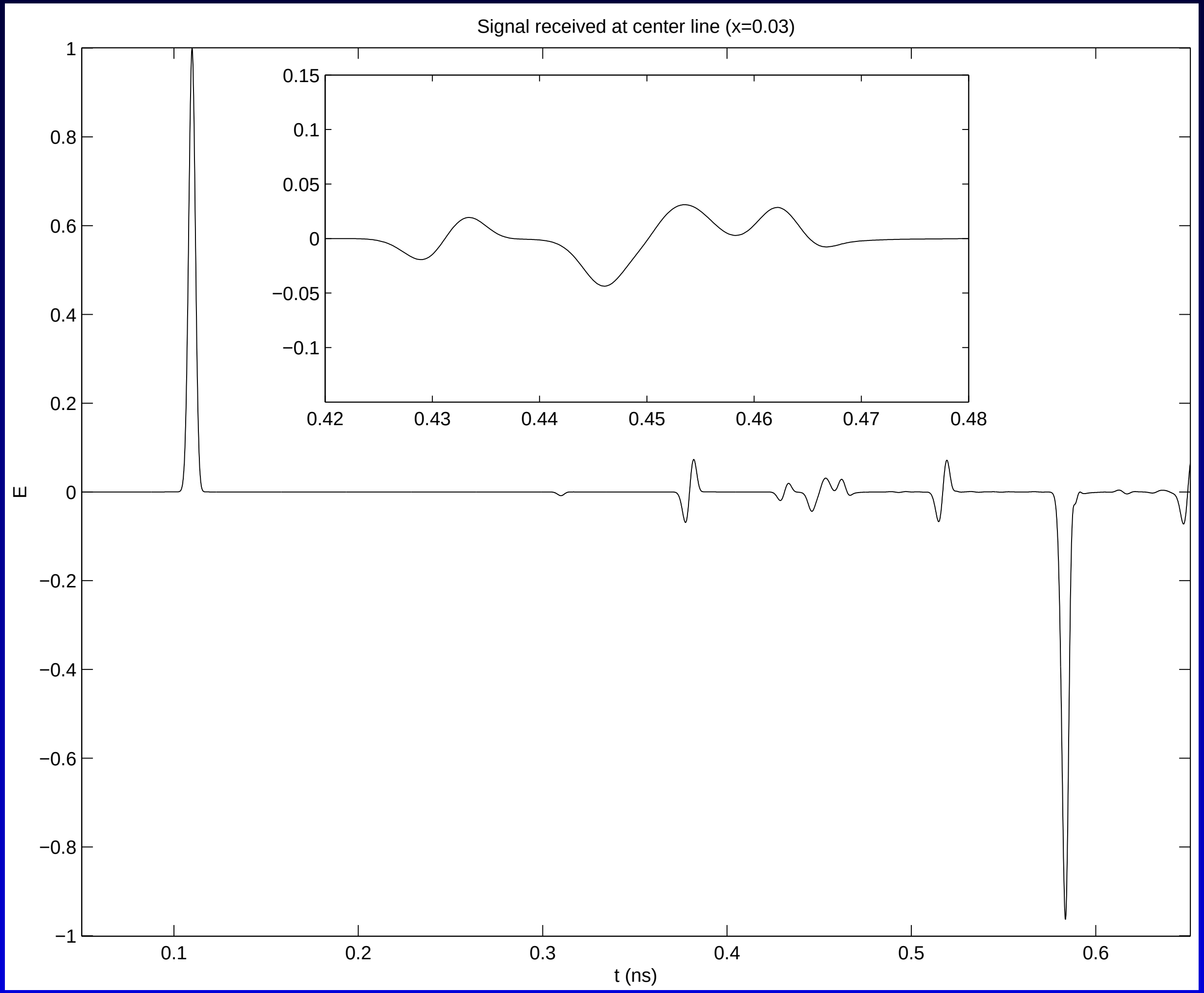
- Second order (piecewise linear) FEM in space
- Second order (centered) FD in time
- Linear solve (sparse)
  - Preconditioned conjugate-gradient (matrix-free)
  - LU factorization
  - **Mass lumping (explicit)**
- Stair-stepping for non-vertical interfaces

Plane Wave Simulation



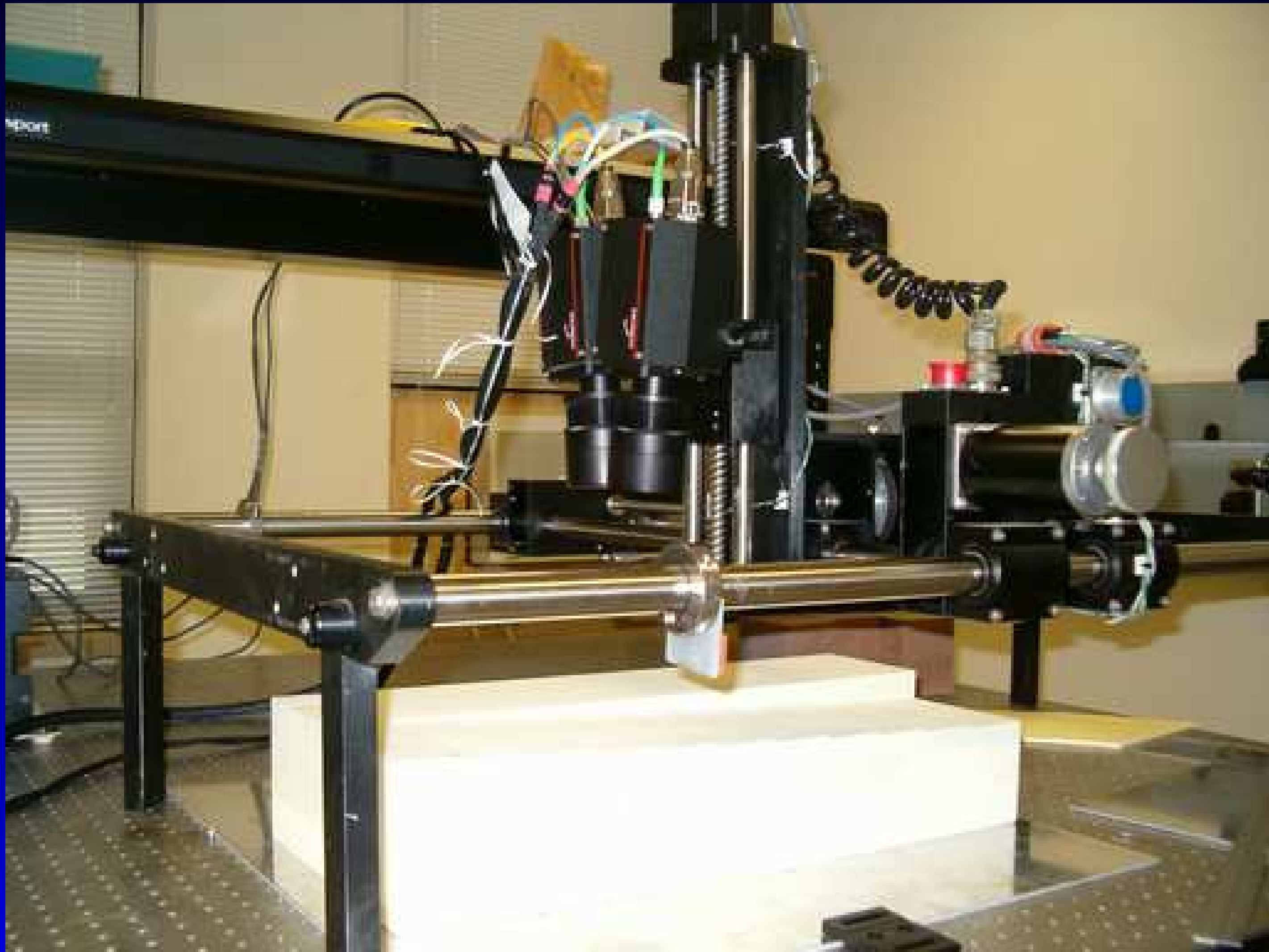
Source located at $x = 0$, receiver at $x = 0.03$.

Plane Wave Signal



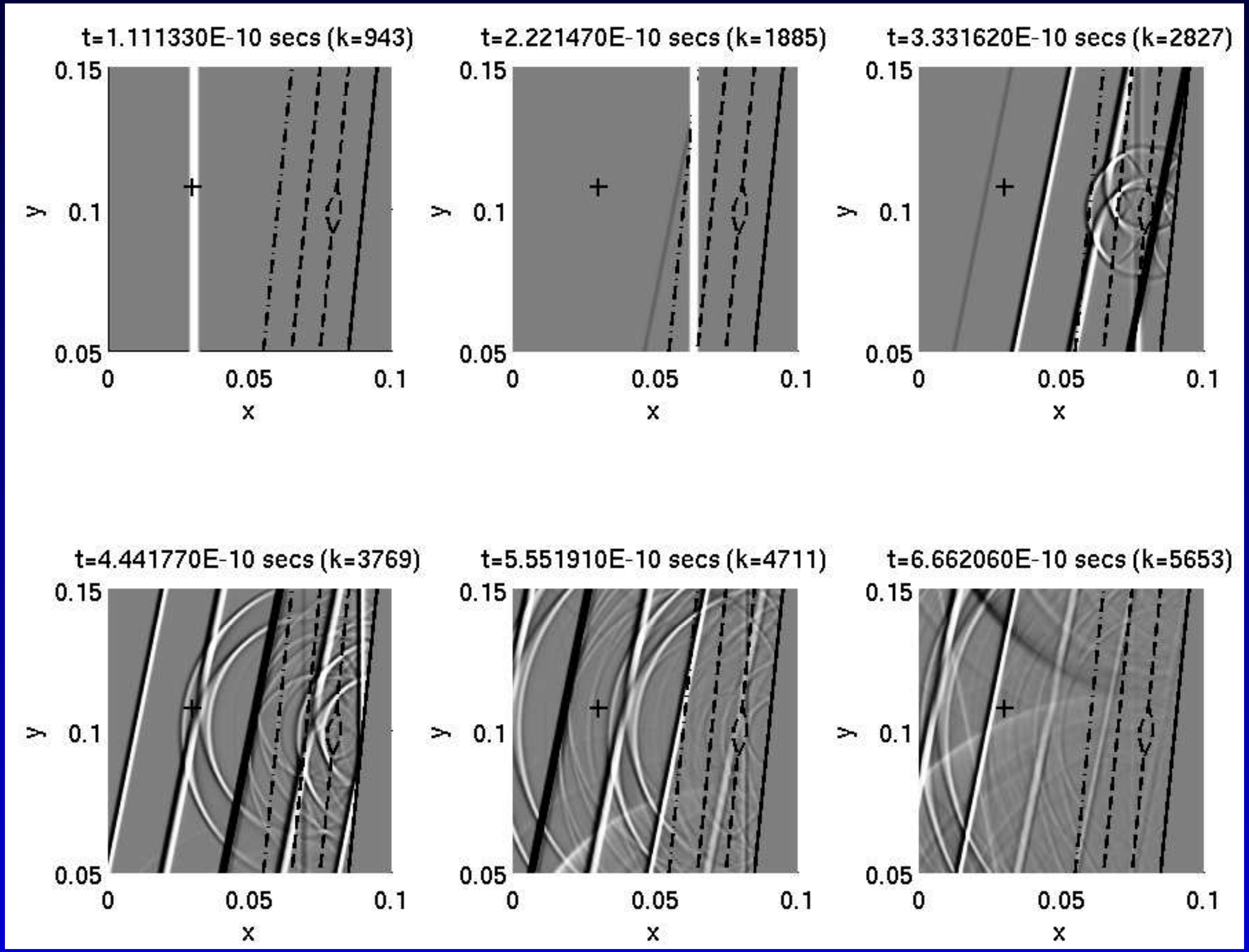
Interrogating signal simulates a sine curve truncated after one half period.
Reflections off void are shown in inset.

Picometrix T-Ray Setup



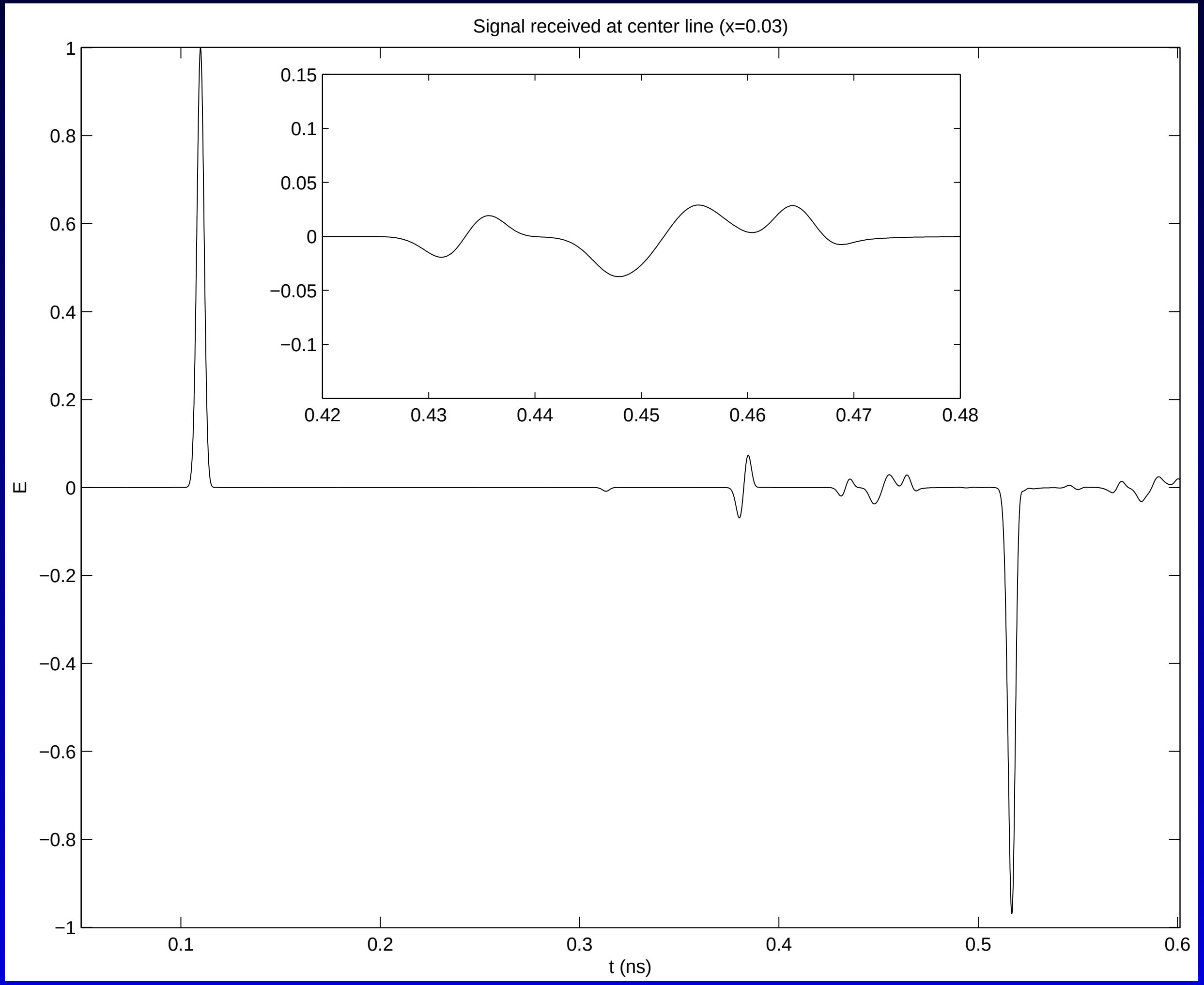
Note the non-normal incidence and ability to focus.

Oblique Plane Wave Simulation



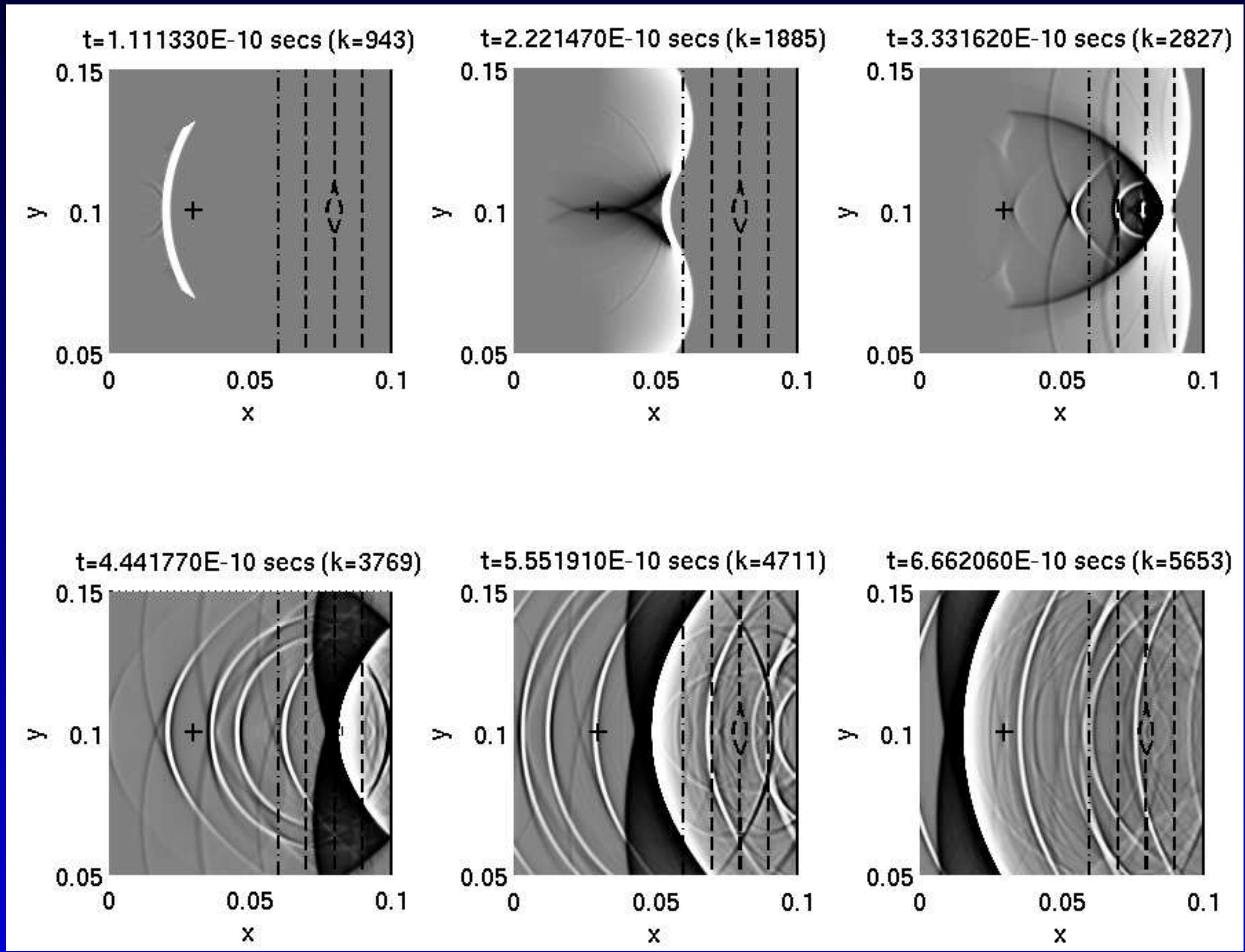
Source located at $x = 0$, receiver at $x = 0.03$, but raised to collect center of plane wave reflection.

Oblique Plane Wave Signal



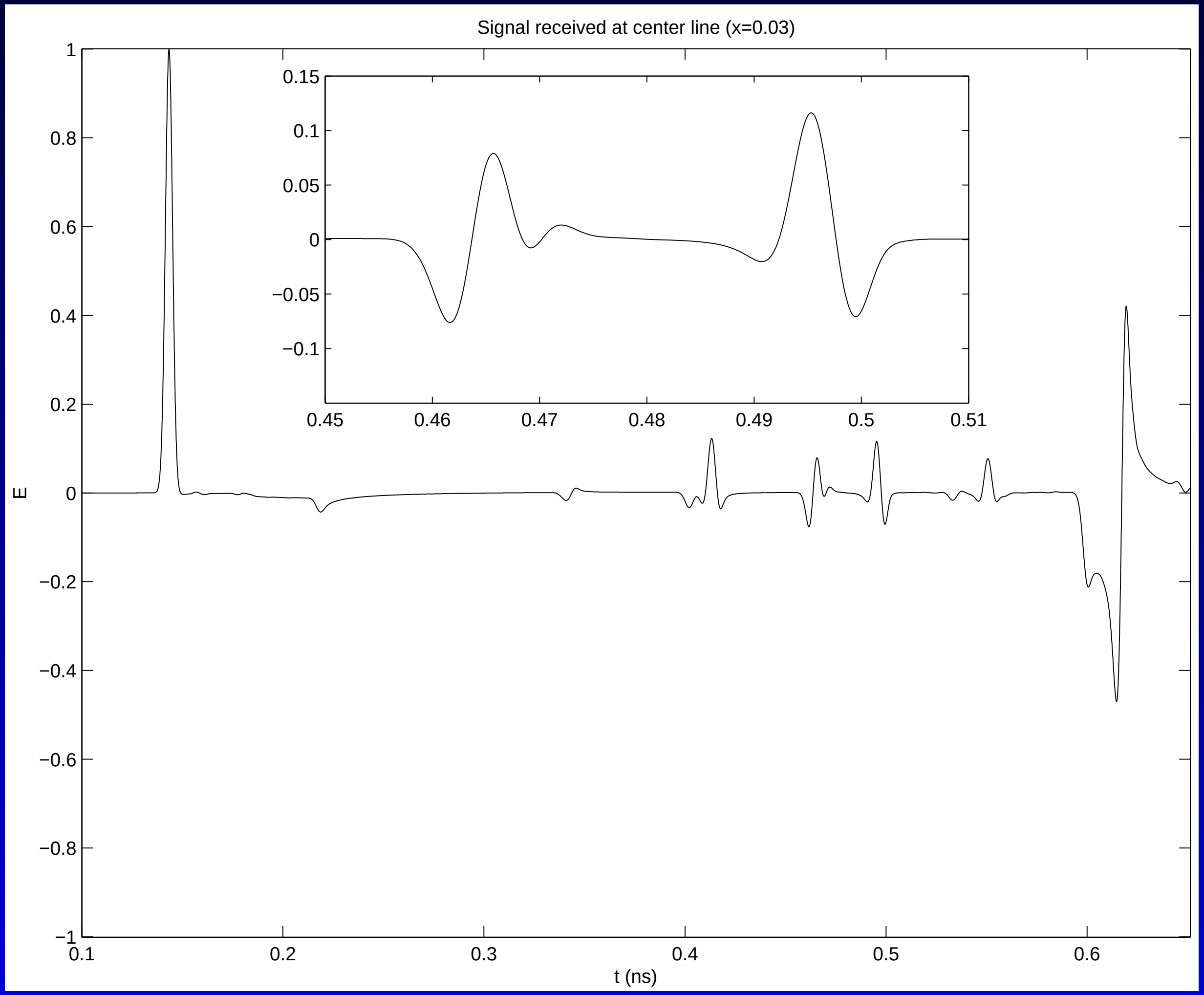
Nearly all of original signal returns even with an oblique angle of incidence. (Note: last knit line removed.)

Focused Wave Simulation



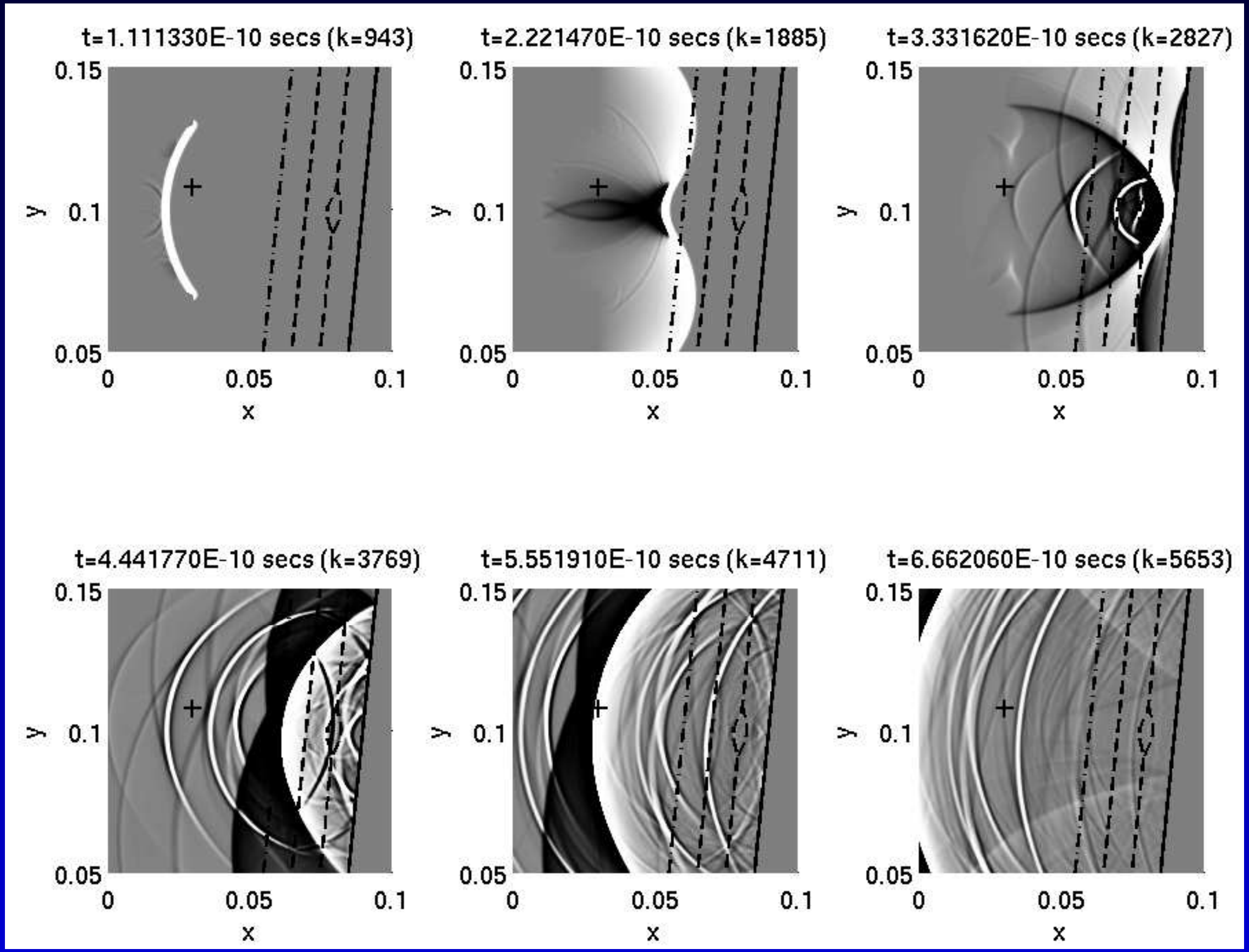
Source modeled using scattered field formulation of point source reflected from elliptical mirror. Receiver located at $x = 0.03$. Note top and bottom boundary conditions are now absorbing.

Focused Wave Signal



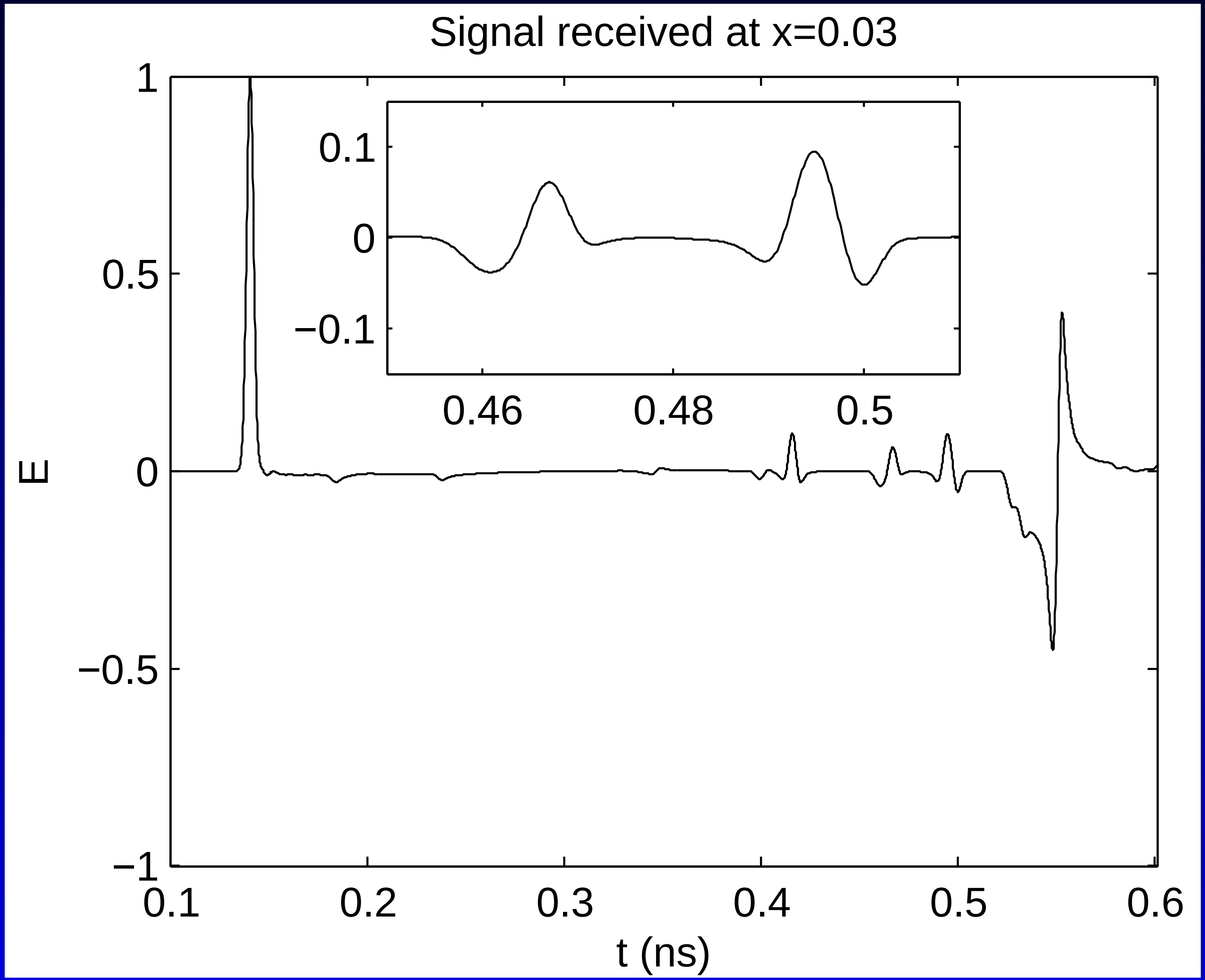
Although reflections off of void are larger, the total energy that returns is less than the plane wave simulation.

Oblique Focused Wave



Source modeled using scattered field formulation of point source reflected from elliptical mirror. Receiver located at $x = 0.03$, but raised to collect center of focused wave reflection.

Oblique Focused Wave Signal



Data received from non-normally incident, focused wave. Reflection from void is similar in magnitude to normal incidence focused wave.

2D Void Inverse Problem

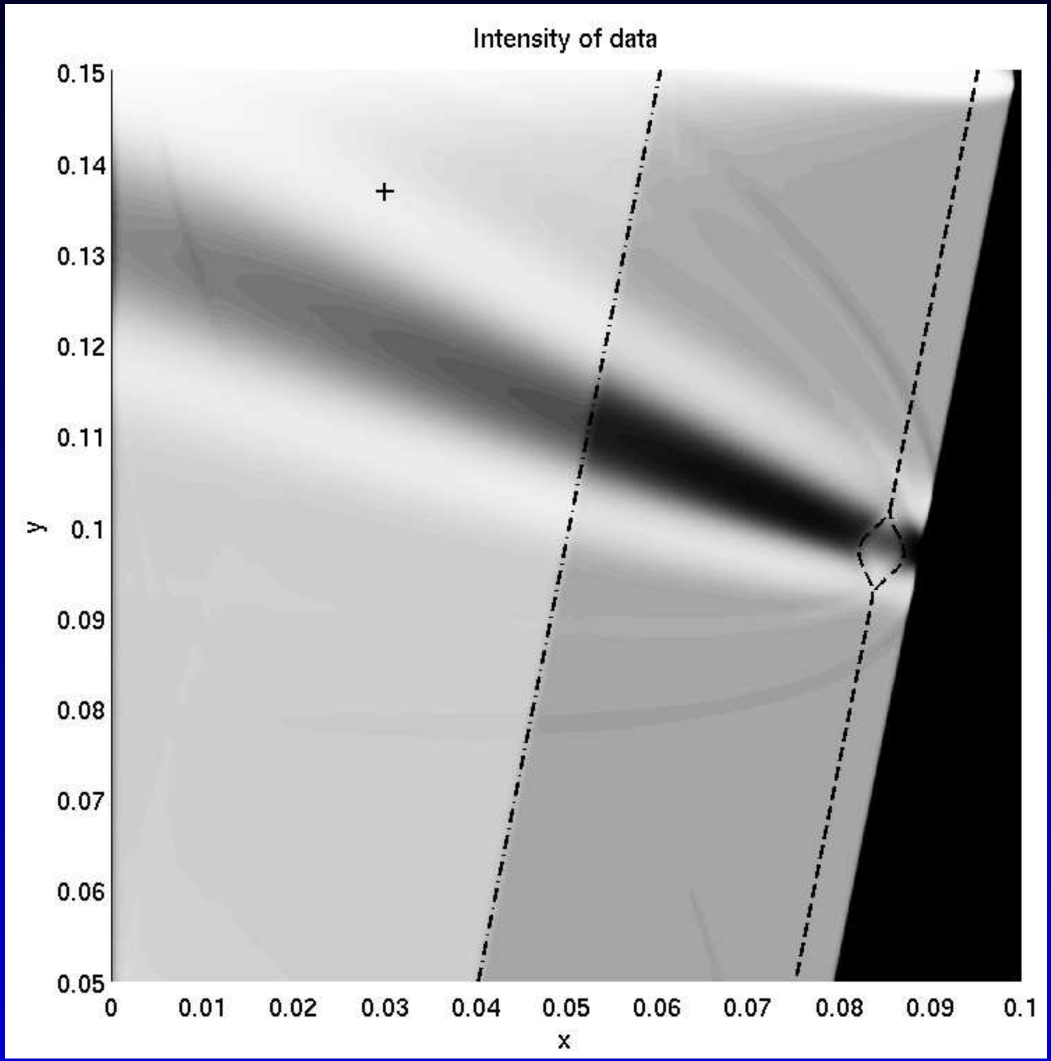
- Assume we have data, $\hat{E}_i$ at times t_i and $\mathbf{x} = \mathbf{x}^+$
- Given the width of an elliptical void w, we can simulate the electric field
- Estimate void width w by solving an inverse problem:

  Find $w \in Q_{ad}$ such that the following objective function is minimized:

$$\mathcal{J}_1(w) = \frac{1}{2S}\sum_{i=1}^{S} |E(t_i, \mathbf{x}^+; w) - \hat{E}_i|^2.$$

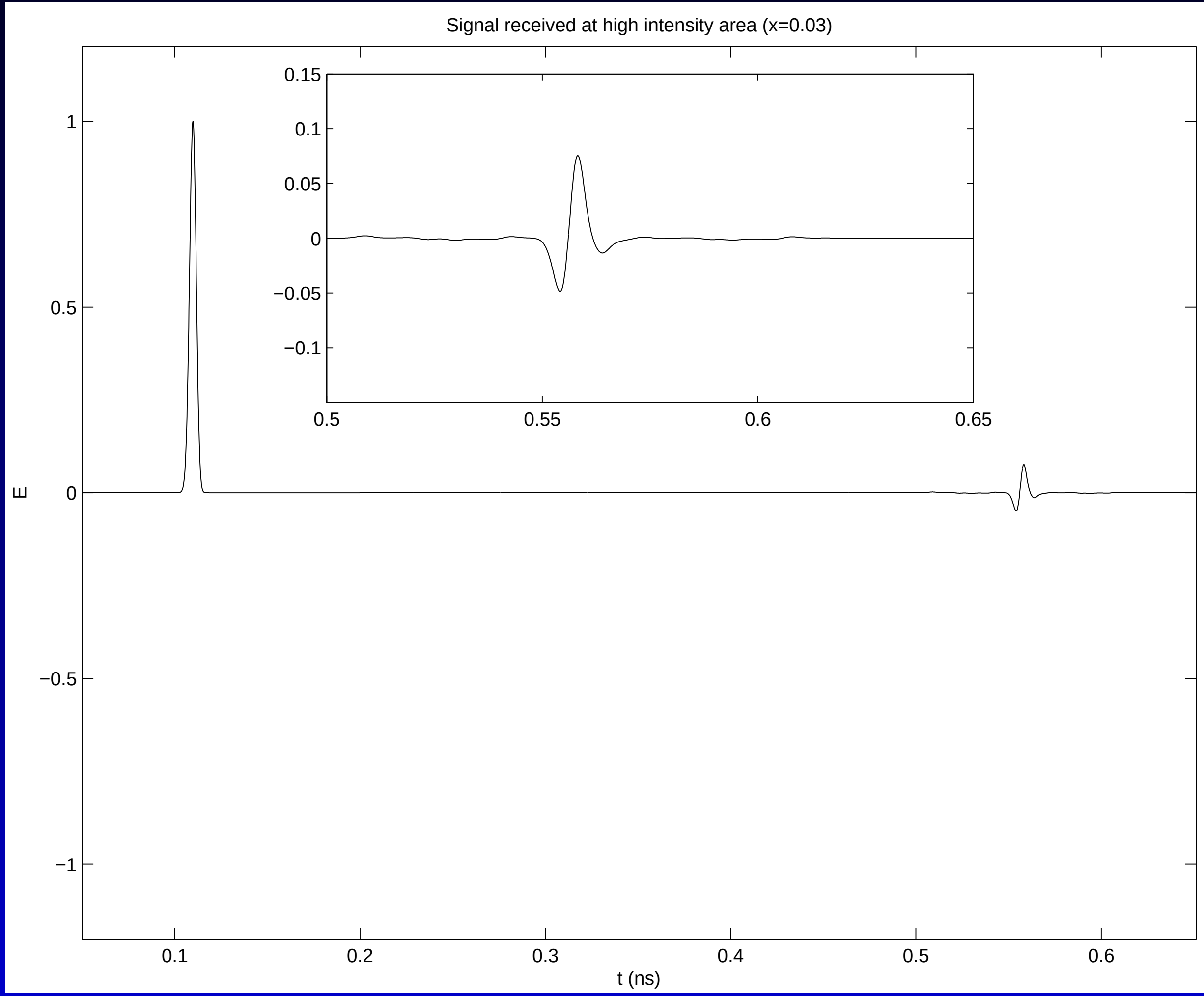
Location of $\mathbf{x}^+$ is crucial.

Intensity of Data Collected



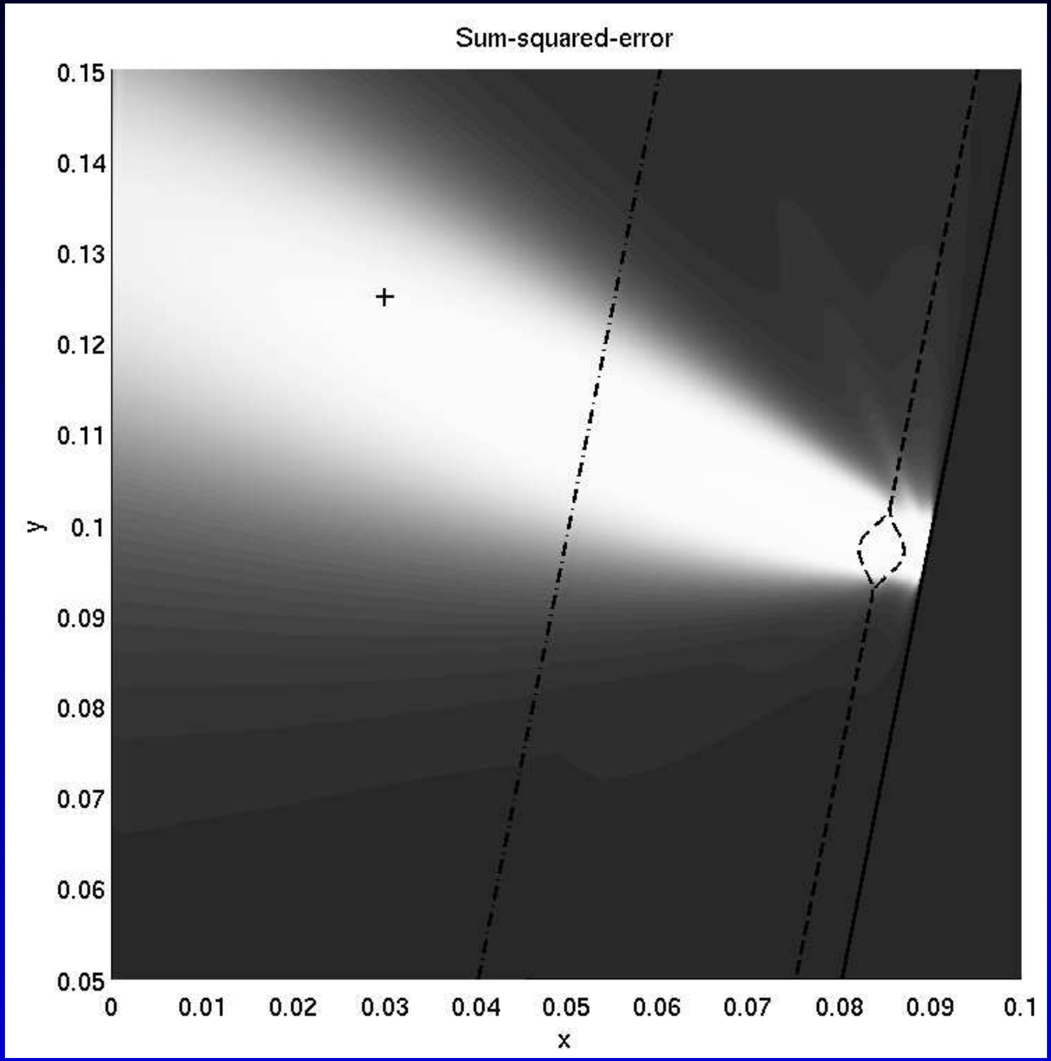
Intensity of data received from non-normally incident, plane wave interrogating material with a void. (Note: knit lines are ignored, dashed line is shown merely to highlight location of void.)

Data Collected at High Intensity



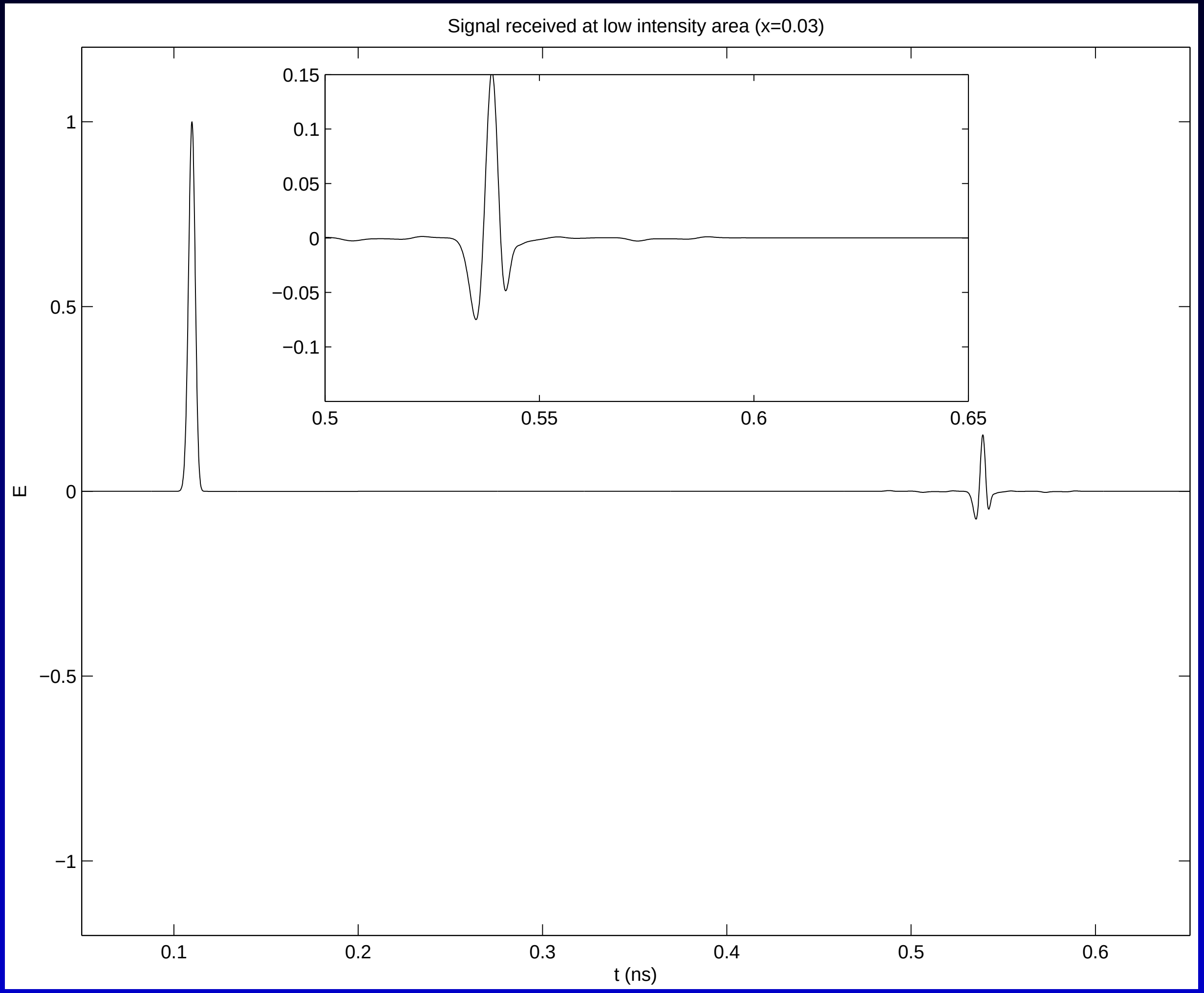
Shown is the difference between data received from non-normally incident, plane wave interrogating material *with and without a void* when receiver placed in high intensity region. (Note: original signal peak is shown for reference.)

Objective Function On Domain



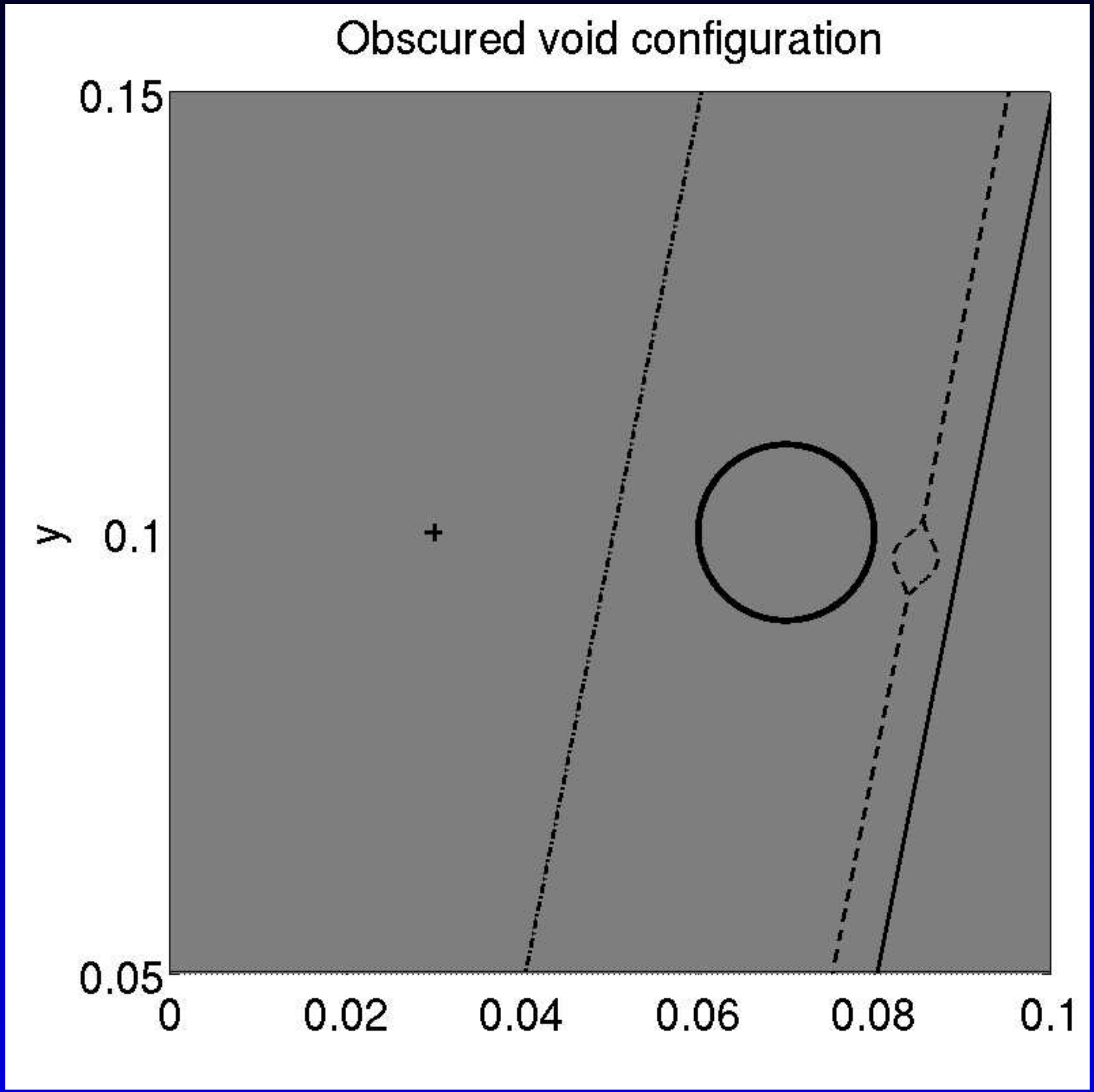
Shown is a surface plot of the objective function at $w = 0$. Cross denotes the location of least intensity in signal received. Note that it correlates with region of greatest contrast between material with void and without.

Data Collected at Low Intensity



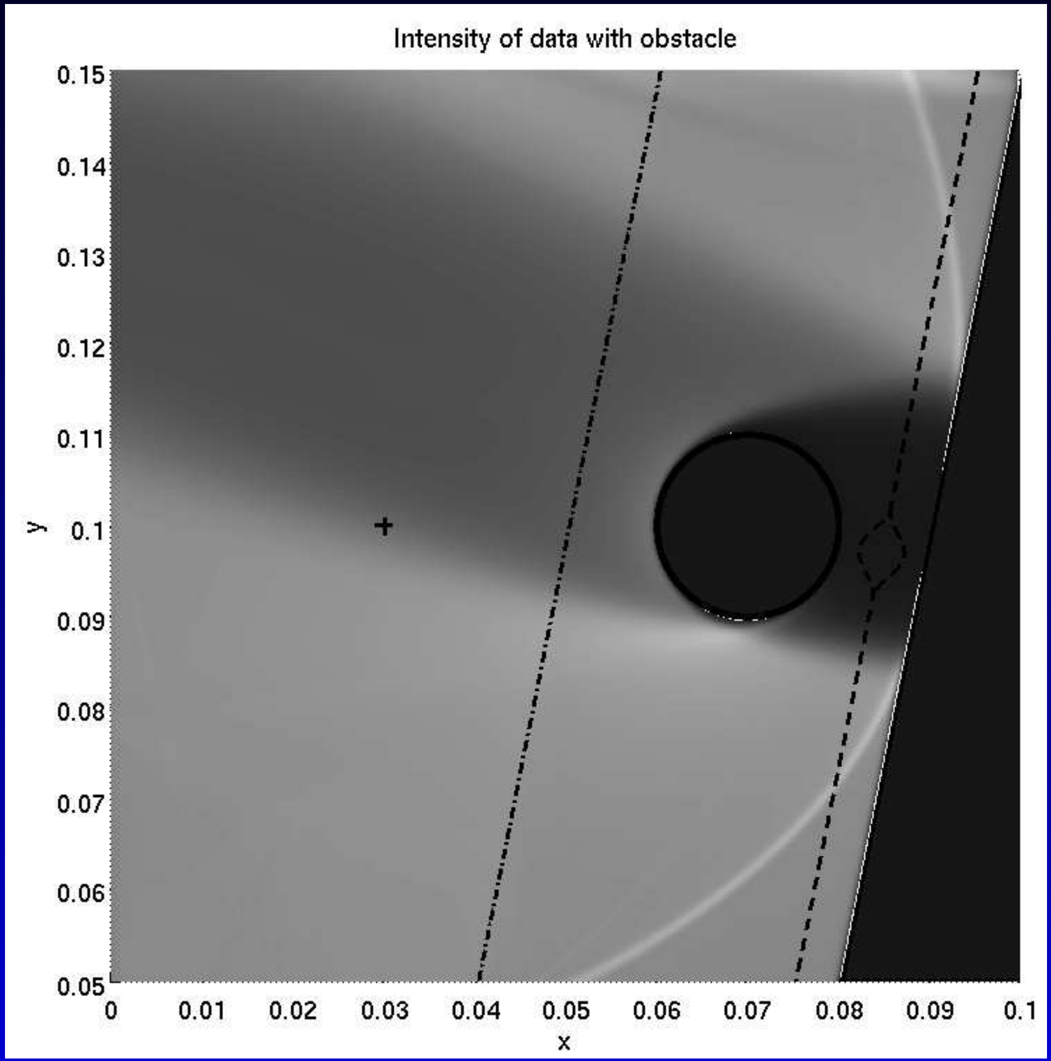
Shown is the difference between data received from non-normally incident, plane wave interrogating material *with and without a void* when receiver placed in low intensity region. Amplitude of difference is doubled.

Domain with Obstacle



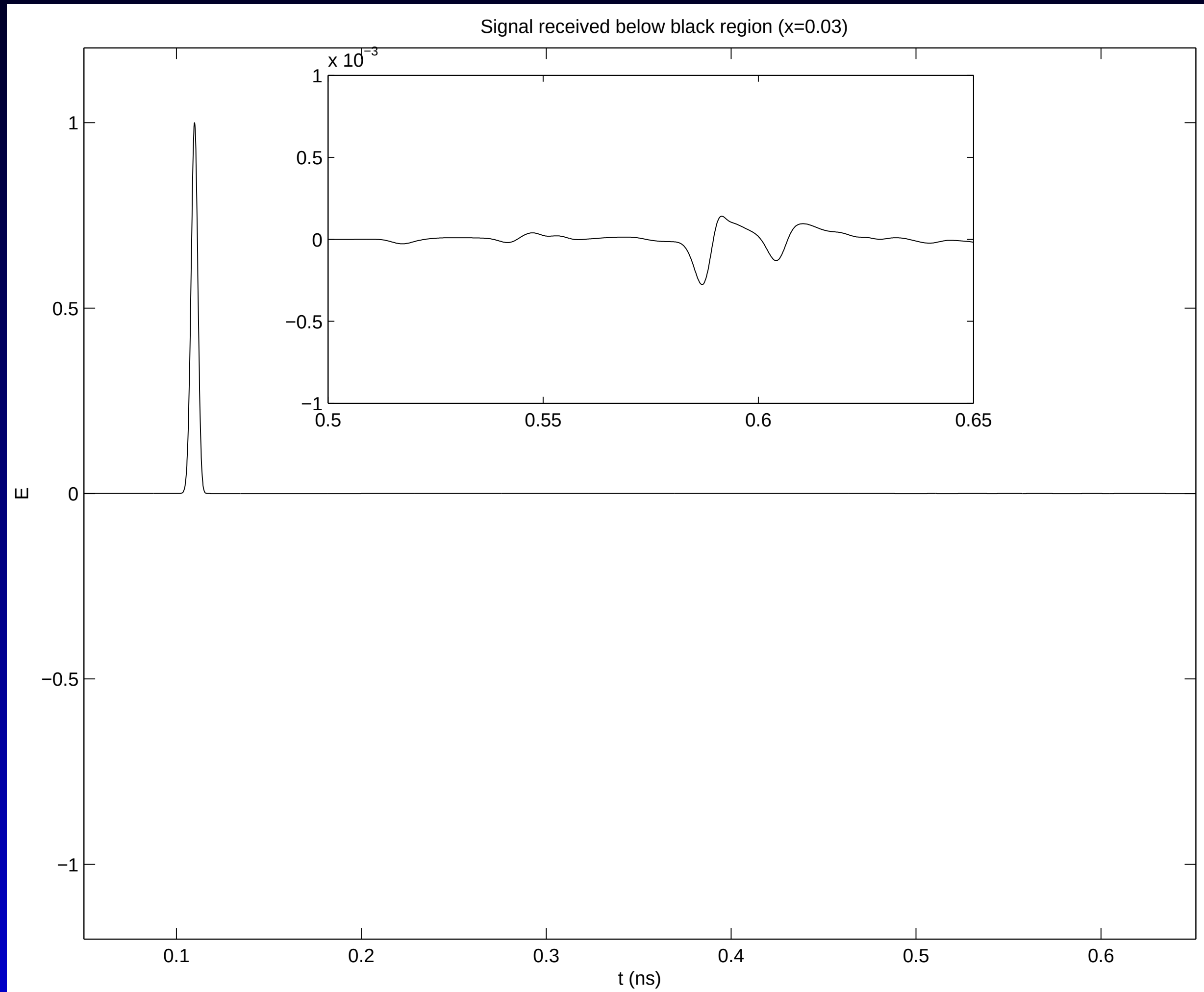
Consider a domain with an opaque obstacle in front of a void.

Intensity of Data Collected



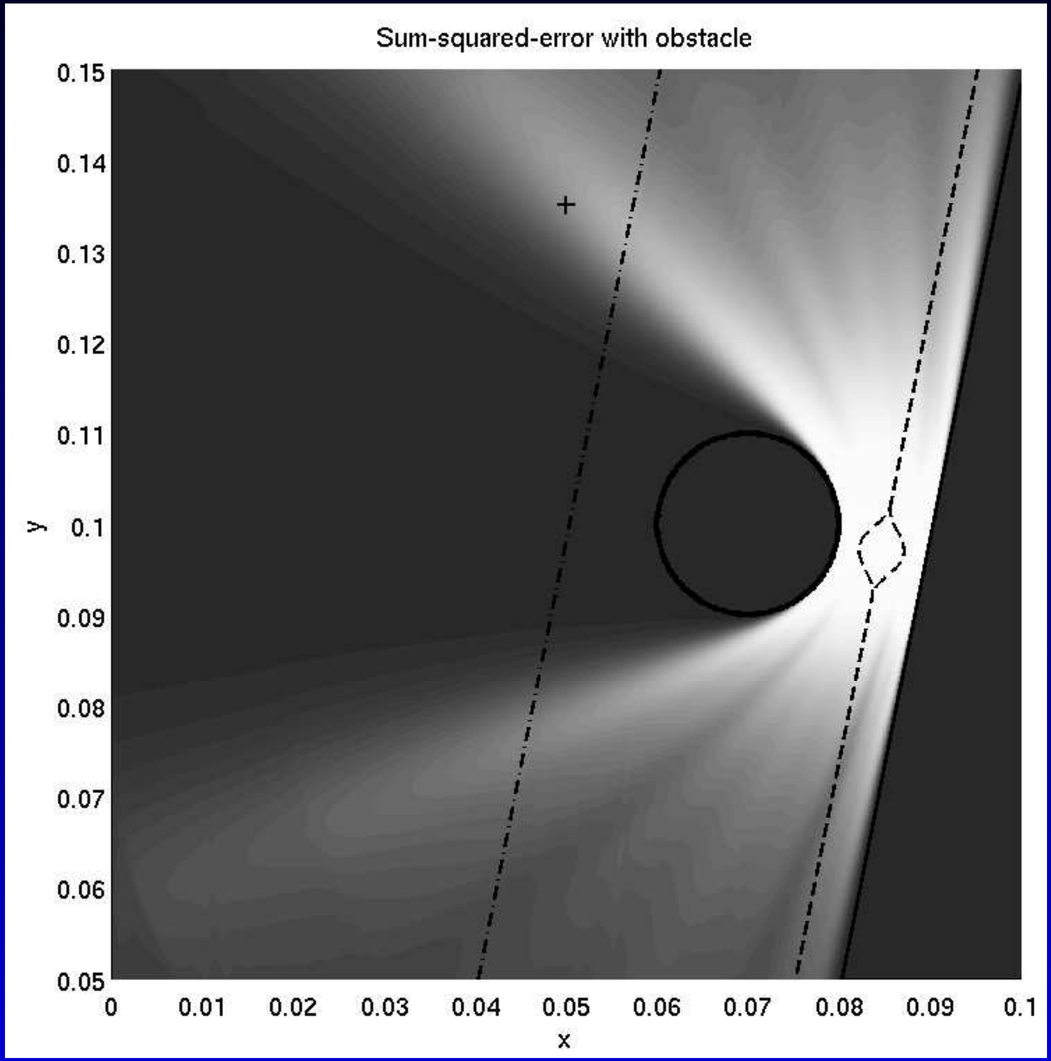
Intensity of data received from interrogating a material with an obstacle in front of a void. Locations for receiver that were considered are directly *above* black area, and directly *below* (i.e., $(.03,.1)$).

Data Collected Below



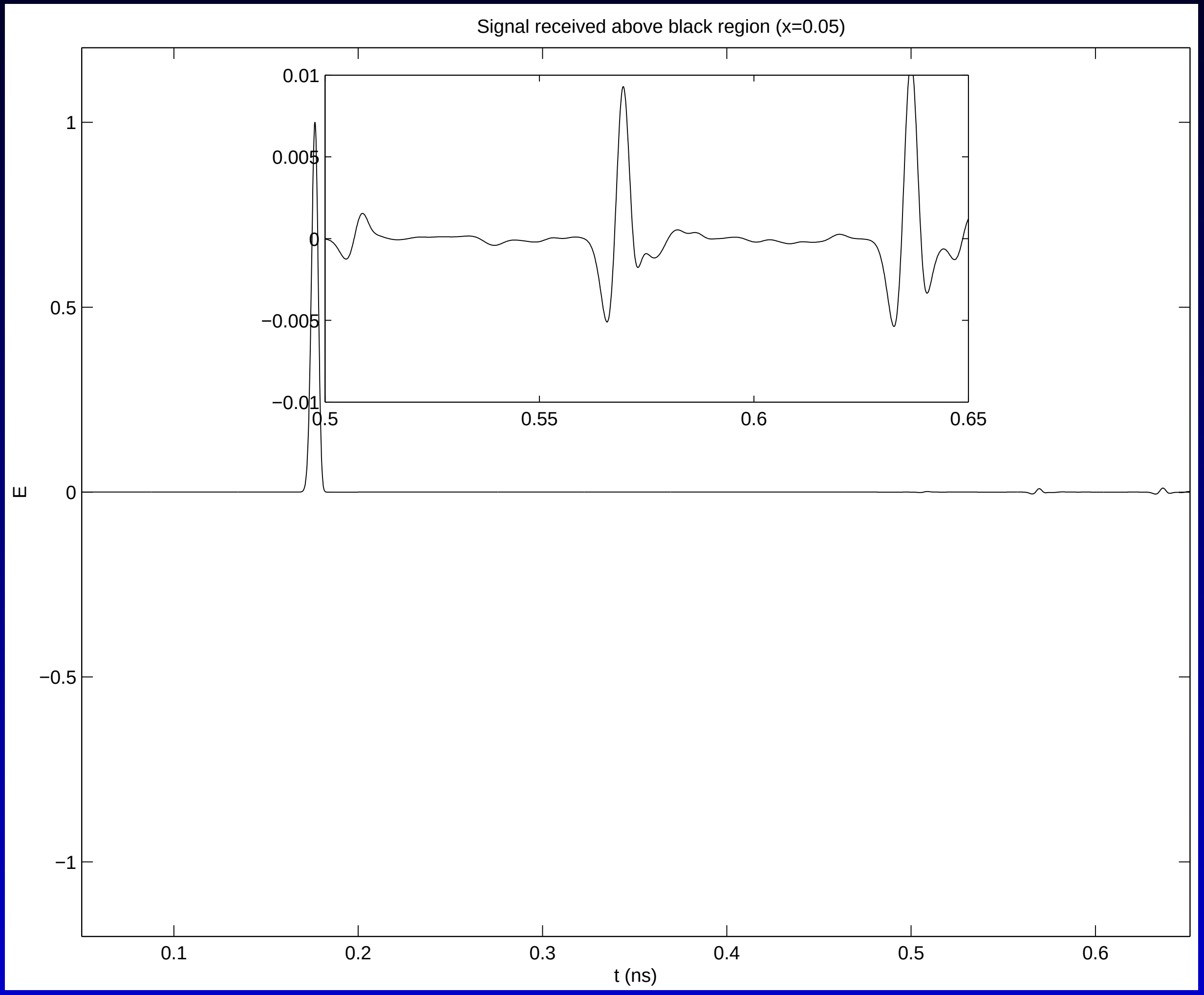
Shown is the difference between data received from non-normally incident, plane wave interrogating material *with and without a void* when receiver placed *below* the invisible region. (Note: original signal peak is shown for reference.)

Objective Function On Domain



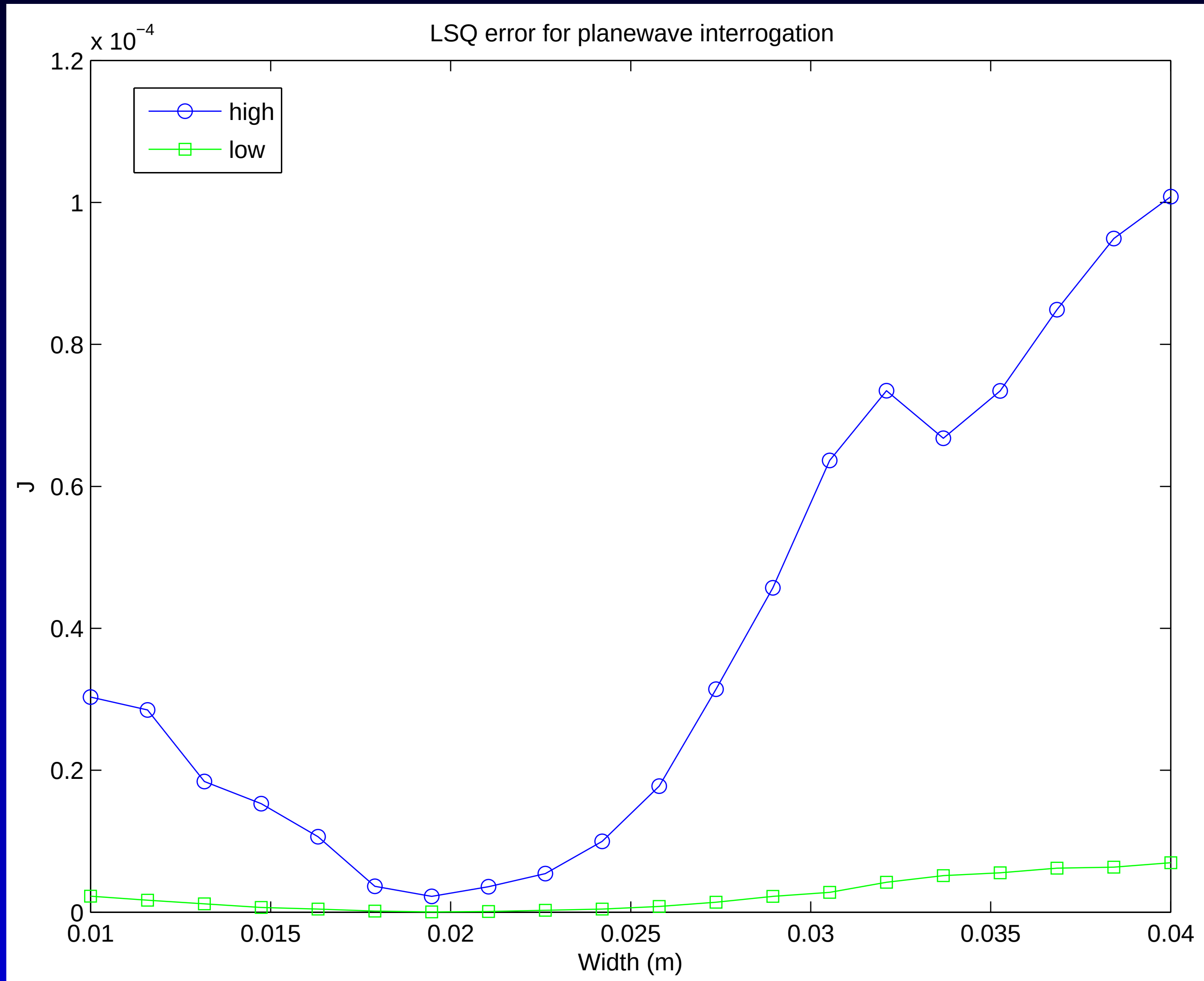
Shown is a surface plot of the objective function at $w = 0$. The cross denotes the location directly *above* the invisible region. Note that it correlates with region of greatest contrast between material with void and without.

Data Collected Above



Shown is the difference between data received from non-normally incident, plane wave interrogating material *with and without a void* when receiver placed *above* invisible region. Amplitude of difference is two orders of magnitude larger.

$\mathcal{J}$ – Objective Function



Objective function versus void size for two locations of receiver: above or below invisible region.

Inverse Problem Results

- Without obstacles, receiver placement should be in region of low intensity for highest contrast (intensity is low due to void).
- With obstacles, receiver placement should be at interface between regions of low and high intensity for highest contrast (intensity is low due to obstacle).
- With proper placement of receiver, LM converges to minimum of $\mathcal{J}$ after 12 iterations
- Each forward solve is 1.5 hours
- This does not incorporate noise (SNR $\approx$ 100:1)

Concluding Remarks

With current power sources and detection devices, reflections from the front surface, voids, and knit lines are difficult to detect. Thus, for now, information is collected from the total reflection off the aluminum backing. More work needs to be done to match simulations to this data, including adding attenuation due to scattering and using data to generate the simulated source.

Continuing Directions

- Modeling Approaches
  - Microscale scattering model
  - Match attenuation observed in data
- Computational Methods
  - Edge elements
  - ABC/PML
  - Faster time-marching (time-splitting)
- Quantify Robustness wrt Uncertainty
  - Material properties
  - Geometry
  - Interrogating signal

References

[BGW05] H. T. Banks, N. L. Gibson and W. P. Winfree, "Gap detection with electromagnetic terahertz signals", *Nonlinear Analysis: Real World Applications* **6**, 381–416, 2005. Tech. Rep. CRSC-TR03-40.

[B-05] H. T. Banks, V. A. Bokil, D. Cioranescu, N. L. Gibson, G. Griso, and B. Miara, "Homogenization of Periodically Varying Coefficients in Electromagnetic Materials", *Journal of Scientific Computing*, **28**:2, 191–221, 2006. Tech. Rep. CRSC-TR05-05.

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[BG06] H. T. Banks and N. L. Gibson, "Void Detection in Foam with Knit Lines Using THz Pulse Interrogation", *Mathematical and Computer Modelling*, **44**:9–10, 807–815, 2006. Tech. Rep. CRSC-TR06-05.

[BBG06] H. T. Banks, V. A. Bokil and N. L. Gibson, "Parameter Estimation Versus Homogenization Techniques in Time-Domain Characterization of Composite Dielectrics", to appear *JIIPP special issue on IPMS-2006*. Tech. Rep. CRSC-TR06-20.

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