Electromagnetic characterization of damage in Space Shuttle foam

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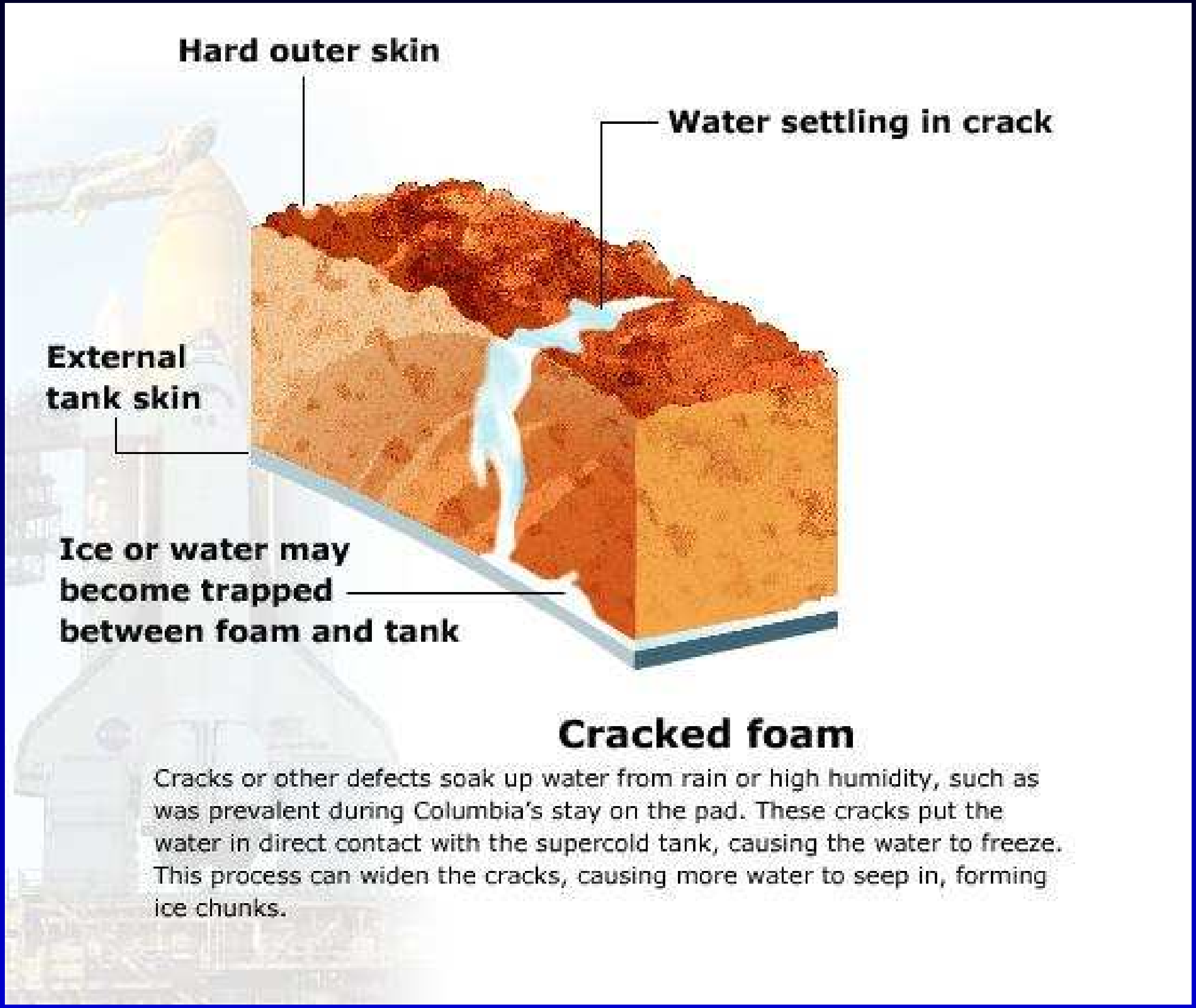
Motivating Application



The particular motivation for this research is the detection of defects in the insulating foam on the space shuttle fuel tanks in order to help eliminate the separation of foam during shuttle ascent.

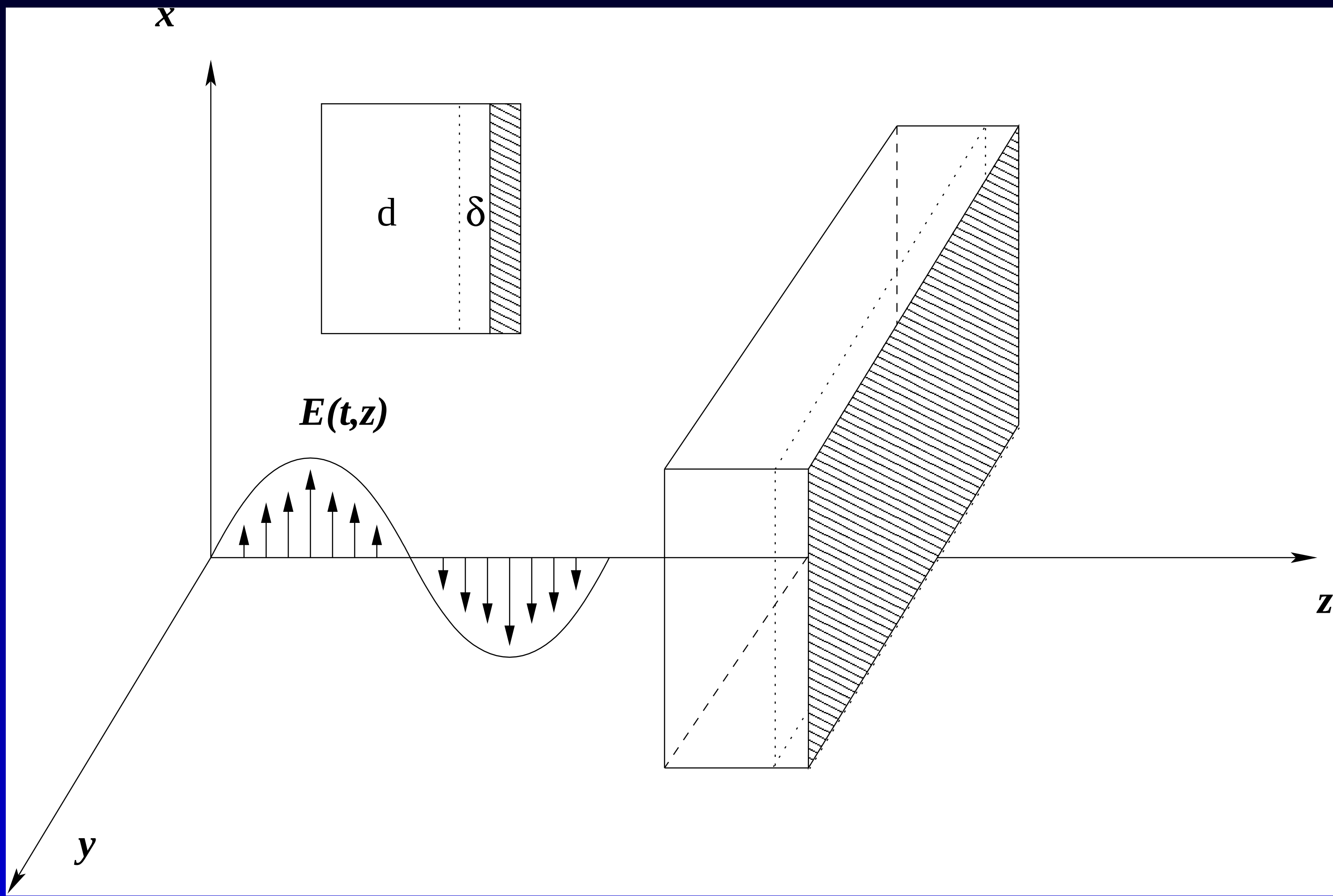
Our Contributions

- **Gap Detection Inverse Problem**
- Modeling Hetereogeneous Materials
  - Distributions of Dielectric Properties
  - Homogenization
- **2D Aspects**
  - **Knit Lines**
  - **Oblique Angles**
  - **Focusing**

Outline 1D Gap Detection

- Model
- Numerical Methods
- Inverse Problem
    - Computational Results
    - Standard Error Analysis

Gap Detection Problem



Model

$$\mu_0\epsilon_0\epsilon_r\ddot{E} + \mu_0 I_\Omega \ddot{P} + \mu_0\sigma I_\Omega \dot{E} - E'' = -\mu_0 \dot{J}_s \qquad \text{in } \Omega \cup \Omega_0$$

$$\tau\dot{P} + P = \epsilon_0(\epsilon_s - \epsilon_\infty)E \qquad \text{in } \Omega$$

$$[\dot{E} - cE']_{z=0} = 0$$

$$[E]_{z=1} = 0$$

$$E(0,z) = \dot{E}(0,z) = 0$$

$$P(0,z) = 0$$

where

$$J_s(t,z) = \delta(z)sin(\omega t)I_{[0,t_f]}(t)$$

and

$$\epsilon_r = (1 + (\epsilon_\infty - 1)I_\Omega).$$

Numerical Discretization

- Second order FEM in space
  - piecewise linear splines
- Second order FD in time
  - Crank-Nicholson (P)
  - Central differences (E)
  - $e_n \longrightarrow p_n \longrightarrow e_{n+1} \longrightarrow p_{n+1} \longrightarrow \cdots$
- E equation implicit, LU factorization used

Finite Element Method in Space

The resulting system of differential equations in semi-discrete form can be written

$$M_1\ddot{e} + M_2\dot{e} + M_3 e + \lambda^2\bar{p} = \eta_0 J \tag{1}$$

$$\dot{\bar{p}} + \lambda\bar{p} = \epsilon_d\lambda M^{\Omega} e. \tag{2}$$

where η_0=$\sqrt{\mu_0/\epsilon_0}$, ϵ_d=$\epsilon_s - \epsilon_\infty$, λ=$1/c\tau$, e and p are vectors representing the approximate values of E and P respectively at the nodes z_i.

$\bar{p}$=$M^{\Omega}p$ where M^{Ω} is the mass matrix integrated only over Ω.

Finite Difference in Time (p)

Our finite difference approximation for (2) is

$$\bar{p}_{n+1} = \bar{p}_n + \frac{\lambda \Delta t}{1 + \lambda \Delta t \theta}(\epsilon_d M^\Omega e_{n+\theta} - \bar{p}_n) \qquad (3)$$

where $[e_n]_j = E(t_n, z_j)$, $[\bar{p}_n]_j = M^\Omega P(t_n, z_j)$, $z_j = jh$.

The value $e_{n+\theta} = \theta e_n + (1-\theta)e_{n+1}$ is a weighted average of e_n and e_{n+1} for relaxation to help with stability of the method.

Note: we take $\theta = 1/2$.

Finite Difference in Time (e)

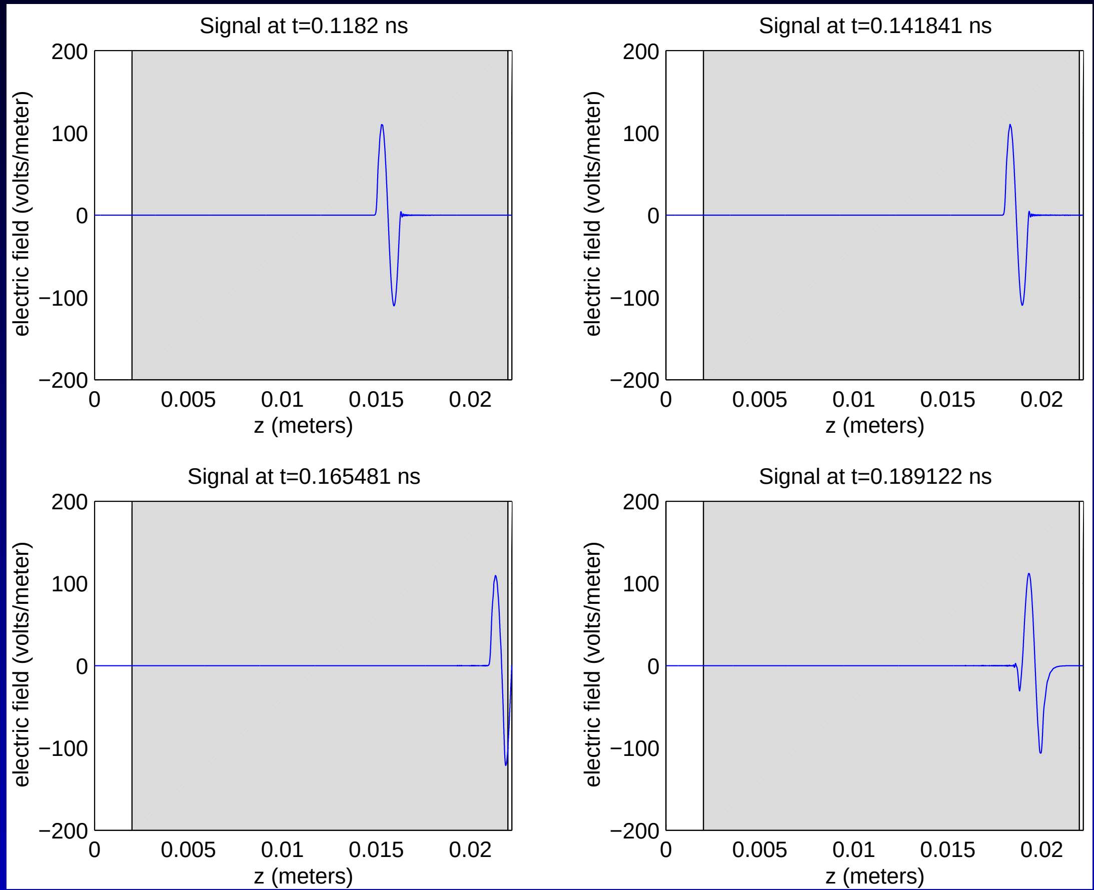
Applying second order central differencing with averaging to (1) gives

$$A_1 e_{n+2} = A_2 e_{n+1} + A_3 e_n + \Delta t^2 \eta_0 J_{n+1} - \lambda^2 \Delta t^2 \bar{p}_{n+1}. \tag{4}$$

As $\bar{p}_{n+1}$ depends explicitly on e_n and e_{n+1}, we could substitute (3) here and have one implicit equation for the update of e.

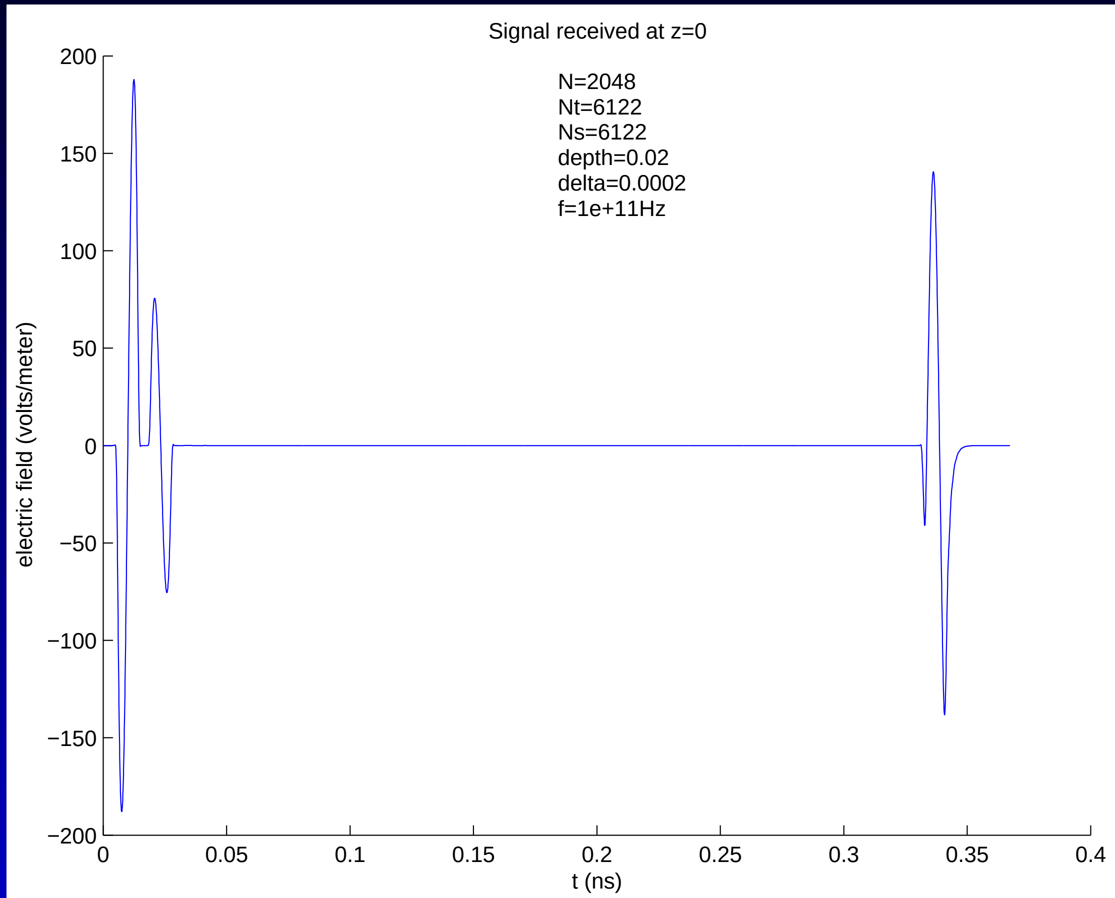
Note: we use LU factorization as A_1 does not change over time.

Sample Problem



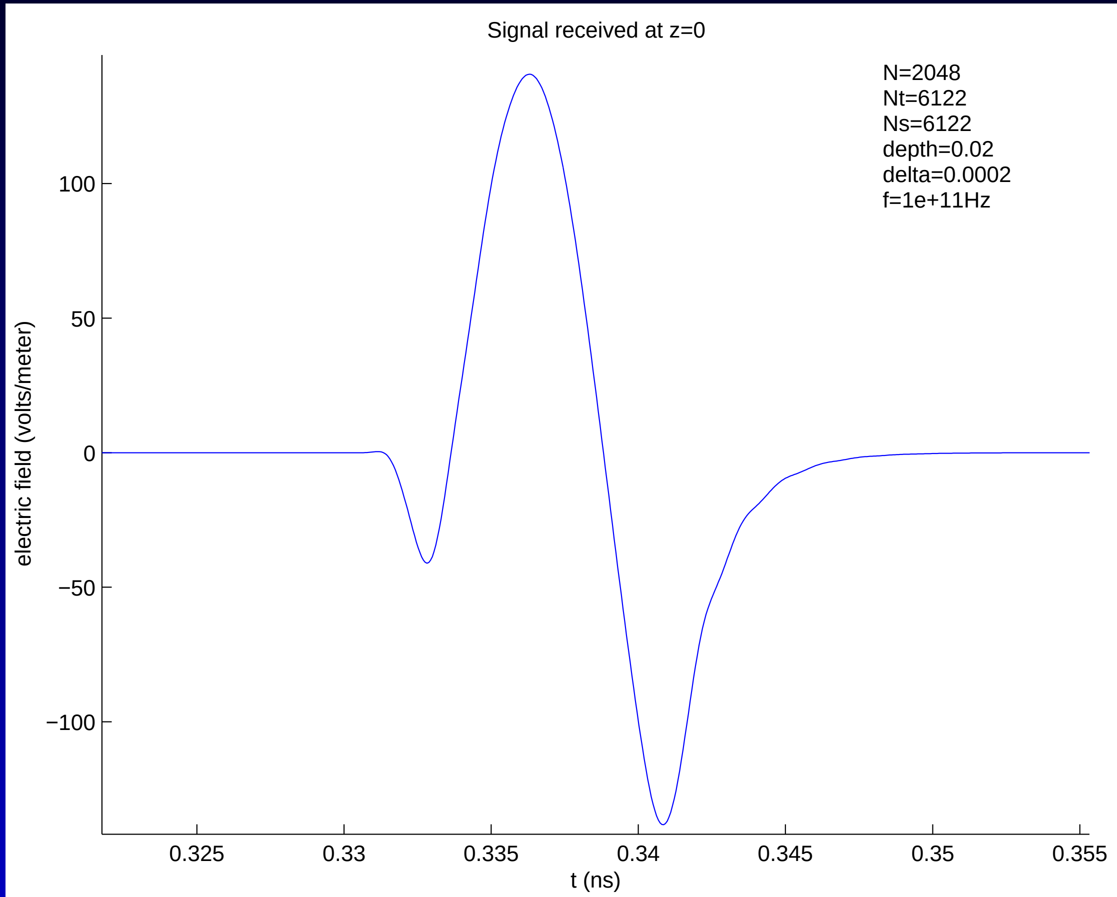
Computed solutions at different times of a windowed electromagnetic pulse at f=$100GHz$ incident on a Debye medium with a crack δ=$.0002m$ wide located d=$.02m$ into the material.

Sample Problem (Cont.)



Reflected signal received at z=0.

Sample Problem (Cont.)



Close up look at reflected signal received at z=0
Shows "important" parts of the signal.

Gap Detection Inverse Problem

- Assume we have data, $\hat{E}_i$, recorded at z=0
- Given d and δ we can simulate the electric field
- Estimate d and δ by solving an inverse problem:

Find q=$(d, \delta) \in Q_{ad}$ such that the following objective function is minimized:

$$\mathcal{J}_1(q) = \frac{1}{2S} \sum_{i=1}^{S} |E(t_i, 0; q) - \hat{E}_i|^2.$$

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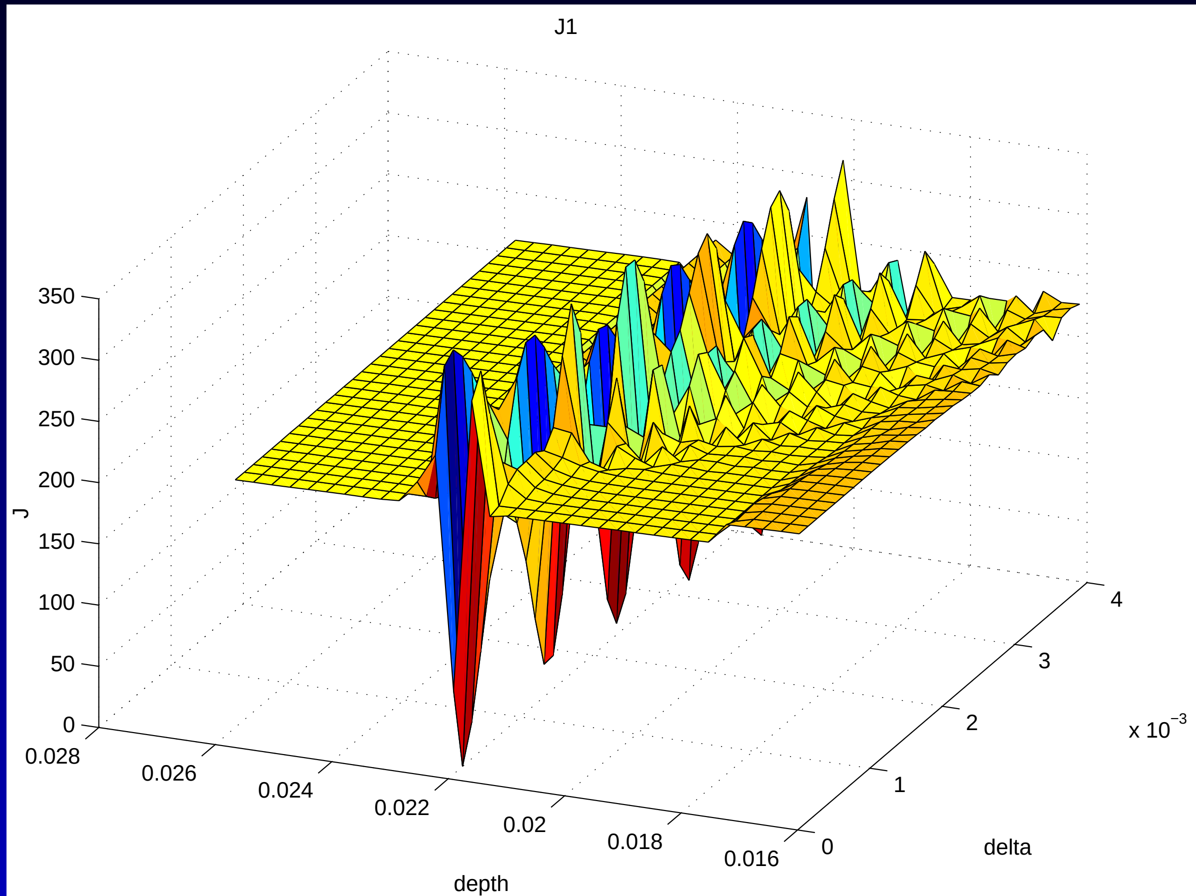
$\mathcal{J}_1(q)$ Surface Plot



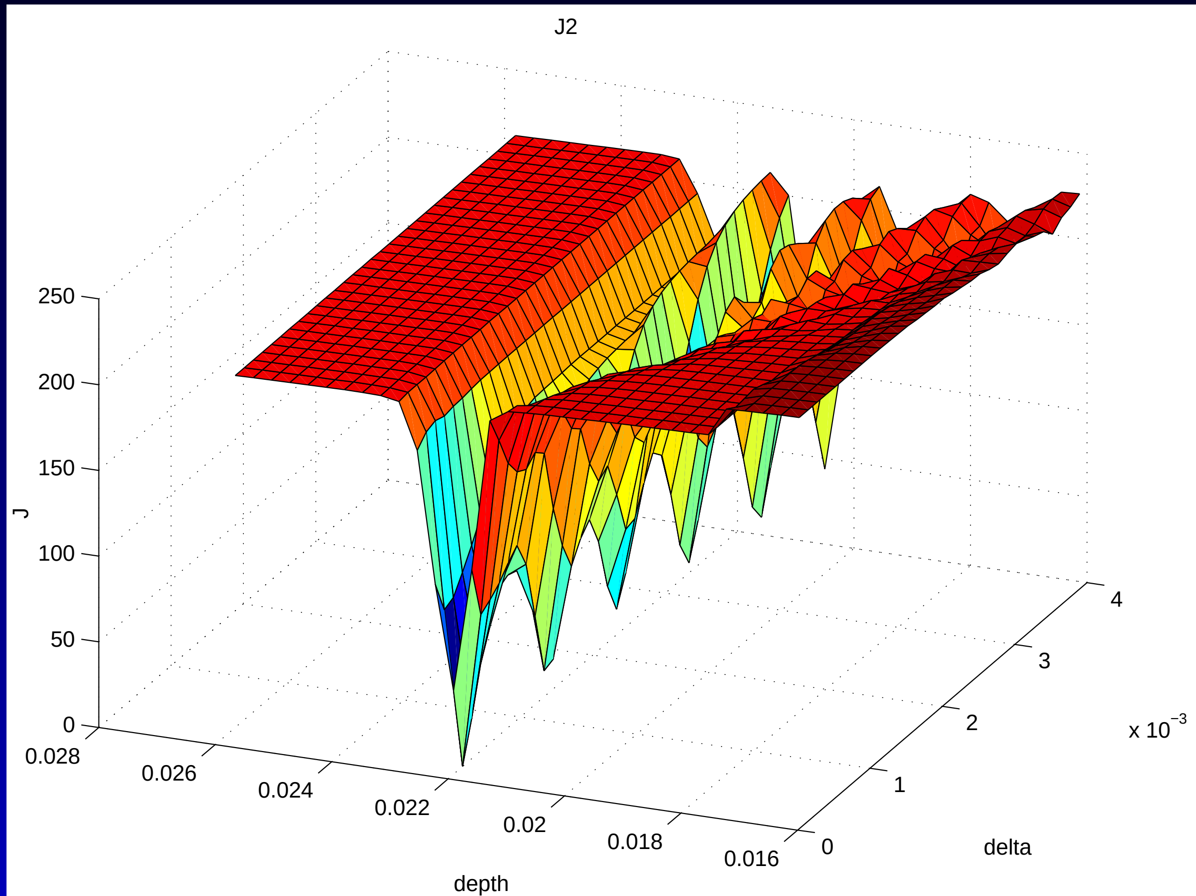
Surface plot of the Ordinary Least Squares objective function demonstrating peaks in $\mathcal{J}_1$, and exhibiting many local minima.

Improved Objective Function

Consider the following formulation of the Inverse Problem:
Find q=$(d, \delta) \in Q_{ad}$ such that the following objective function is minimized:

$$\mathcal{J}_2(q) = \frac{1}{2S} \sum_{i=1}^{S} \Big| |E(t_i, 0; q)| - |\hat{E}_i| \Big|^2 .$$

$\mathcal{J}_2(q)$ Surface Plot



Close up surface plot of our Modified Least Squares objective function demonstrating lack of peaks in $\mathcal{J}_2$, but still exhibiting many local minima.

Check Point Method

The diagonal "trench" occurs approximately along the line

$$d = -\frac{1}{\sqrt{\epsilon_0}}(\delta - \delta^*) + d^*.$$

Also, the minima occur every $\frac{\lambda}{4}m$ along this line. Therefore, if our optimization routine detects a local minima, we test $\frac{\lambda}{4}$ on either side of the local minima to see if there is a smaller minima nearby. If so, we restart our optimizer at the new smallest point.

Levenberg-Marquardt Method

We re-write the objective function as

$$J(q) = \frac{1}{2S} R^T R$$

where R_i=$(|E(t_i, 0; q)| - |\hat{E}_i|)$ is the residual. To update our approximation to q we make the Gauss-Newton update step q_+=$q_c + s_c$ where

$$s_c = -\left(R'(q_c)^T R'(q_c) + \nu_c I\right)^{-1} R'(q_c)^T R(q_c).$$

is the step, q_c is the current approximation, and q_+ is the resulting approximation. The value ν_c is called the Levenberg-Marquardt parameter.

Final Estimates (d)

d		δ = .0002	.0004	.0008
.02	(N=1024)	.0200022	.0200006	.0200002
.04	(N=2048)	.0399974	.0400005	.0399999
.08	(N=4096)	.0799987	.0800006	.0800003
.1	(N=8192)	.0999974	.1	.0999999
.2	(N=16384)	.200005	.2	.200001

Final Estimates (δ)

		δ		
d		.0002	.0004	.0008
.02	(N=1024)	.000196754	.000398642	.00079707
.04	(N=2048)	.000203916	.000394204	.000793622
.08	(N=4096)	.000202273	.000395791	.000794401
.1	(N=8192)	.000203876	.000396203	.000795985
.2	(N=16384)	.000191808	.00040297	.00080129

Random Noise

We add random noise to the data signal in order to more closely simulate the experimental process in data collection. We define

$$\hat{E}_i = E_i + \beta \nu_r \eta_i,$$

where

$$\eta_i \sim \mathcal{N}(0, 1).$$

The coefficient $\beta = \max_i \hat{E}_i / 10$ is just a scaling factor (to aid in comparison to relative noise), and ν_r is what we call the noise level.
Note that the variance $\sigma^2 = \beta^2 \nu_r^2$ is constant.

Standard Error Analysis

As $N_s \to \infty$, we have that

$$\hat{q}_{OLS} \sim \mathcal{N}_2 \left(q_0, \sigma_0^2 \left[\mathcal{E}^T(q_0) \mathcal{E}(q_0) \right]^{-1} \right)$$

where $\mathcal{E}(\hat{q})=\frac{\partial |E|}{\partial q}(\hat{q})$ is an $N_s \times 2$ matrix. Also, the scale parameter σ_0^2 is approximately given by

$$\sigma_0^2 = \frac{1}{N_s - 2} \sum_{i=1}^{N_s} \left(|E(t_i, 0; q_0)| - |\hat{E}_i| \right)^2$$

and q_0 denotes the theoretical “true” value for the system which is being observed (generally unknown).

Estimating Covariance

Since q_0 is unknown, we approximate our covariance matrix using our estimate $\hat{q}_{OLS}$:

$$C = \sigma^2_{OLS} \left[\mathcal{E}^T(\hat{q}_{OLS}) \mathcal{E}(\hat{q}_{OLS}) \right]^{-1},$$

where

$$\sigma^2_{OLS} = \frac{1}{N_s - 2} \sum_{i=1}^{N_s} \left(|E(t_i, 0; \hat{q}_{OLS})| - |\hat{E}_i| \right)^2.$$

The standard errors for q_k are then given by $\sqrt{C_{kk}}$.

Computing Covariance

In order to compute the partial derivatives with respect to d and δ in $\mathcal{E}$ we employ finite differencing, which requires an additional forward simulation for each q_j. For example:

$$\mathcal{E}_{i1} = \frac{\partial |E|}{\partial q_1}(t_i, 0; \hat{q})$$
$$\approx \frac{|E\,(t_i, 0; [\hat{q}_1, \hat{q}_2])| - |E(t_i, 0; [(1-h_d)\hat{q}_1, \hat{q}_2])|}{h_d \hat{q}_1}$$

and similarly for each $\mathcal{E}_{i2}$.

Confidence Intervals Example

In the case of d^*=.02, δ^*=.0002 and with ν_r=.01 our covariance matrix is

$$C = \begin{bmatrix} 2.37122 \times 10^{-15} & -4.43815 \times 10^{-15} \\ -4.43815 \times 10^{-15} & 9.1829 \times 10^{-15} \end{bmatrix},$$

which results in the confidence intervals

$$d \in (2.00004 \pm 4.86952 \times 10^{-6}) \times 10^{-2}$$
$$\delta \in (1.9941 \pm 0.000958274) \times 10^{-4}.$$

Confidence Intervals for d $(\nu_r = 0)$

δ	$d^* = .02\ (N = 2048)$
$.0002$	$(2.00005 \pm 9.30284 \times 10^{-7}) \times 10^{-2}$
$.0004$	$(2.00001 \pm 6.50411 \times 10^{-7}) \times 10^{-2}$
$.0008$	$(2.00001 \pm 4.91232 \times 10^{-7}) \times 10^{-2}$

δ	$d^* = .04\ (N = 4096)$
$.0002$	$(4.00013 \pm 1.62162 \times 10^{-6}) \times 10^{-2}$
$.0004$	$(4.00001 \pm 1.19064 \times 10^{-6}) \times 10^{-2}$
$.0008$	$(4.00002 \pm 9.05240 \times 10^{-7}) \times 10^{-2}$

Confidence intervals for the OLS estimate of d when the data is generated with no noise (i.e., ν_r=0.0).

Confidence Intervals for d $(\nu_r = .1)$

δ	$d^* = .02\ (N = 2048)$
.0002	$(2.00000 \pm 4.72903 \times 10^{-5}) \times 10^{-2}$
.0004	$(2.00003 \pm 3.39327 \times 10^{-5}) \times 10^{-2}$
.0008	$(2.00003 \pm 2.79911 \times 10^{-5}) \times 10^{-2}$

δ	$d^* = .04\ (N = 4096)$
.0002	$(4.00014 \pm 5.48283 \times 10^{-5}) \times 10^{-2}$
.0004	$(4.00002 \pm 3.87474 \times 10^{-5}) \times 10^{-2}$
.0008	$(4.00003 \pm 3.19526 \times 10^{-5}) \times 10^{-2}$

Confidence intervals for the OLS estimate of d when the data is generated with noise level ν_r=0.1.

Confidence Intervals for δ $(\nu_r = 0)$

δ	$d^* = .02$ $(N = 2048)$
$.0002$	$(1.99272 \pm 0.000182978) \times 10^{-4}$
$.0004$	$(4.00035 \pm 0.000201885) \times 10^{-4}$
$.0008$	$(7.99833 \pm 0.000136586) \times 10^{-4}$

δ	$d^* = .04$ $(N = 4096)$
$.0002$	$(1.98142 \pm 0.000317616) \times 10^{-4}$
$.0004$	$(4.00737 \pm 0.000369841) \times 10^{-4}$
$.0008$	$(8.00332 \pm 0.000251291) \times 10^{-4}$

Confidence intervals for the OLS estimate of δ when the data is generated with no noise (i.e., ν_r=0.0).

Confidence Intervals for δ $(\nu_r = .1)$

δ	$d^* = .02\ (N = 2048)$
$.0002$	$(2.00017 \pm 0.00932701) \times 10^{-4}$
$.0004$	$(4.00070 \pm 0.0105331) \times 10^{-4}$
$.0008$	$(7.99698 \pm 0.00778563) \times 10^{-4}$

δ	$d^* = .04\ (N = 4096)$
$.0002$	$(1.97674 \pm 0.0107203) \times 10^{-4}$
$.0004$	$(4.01229 \pm 0.0120445) \times 10^{-4}$
$.0008$	$(8.00361 \pm 0.00886925) \times 10^{-4}$

Confidence intervals for the OLS estimate of δ when the data is generated with noise level ν_r=0.1.

Comments on 1D Gap Problem

- Our modified Least Squares objective function "fixes" peaks in $\mathcal{J}$
- Can test on both sides of detected minima to ensure global minimization
- We are able to detect a $.2mm$ wide crack behind a $20cm$ deep slab
- Even adding random noise (equivalent to 20% relative noise) does not significantly hinder our inverse problem solution method, and only slightly broadens the confidence intervals in a sensitivity analysis

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2D Problem Outline

- Motivation
- Model
- Parameter Identification
  - Clausius-Mossotti
  - Experimental Data
- Computational Methods
- Simulations
- Inverse Problem

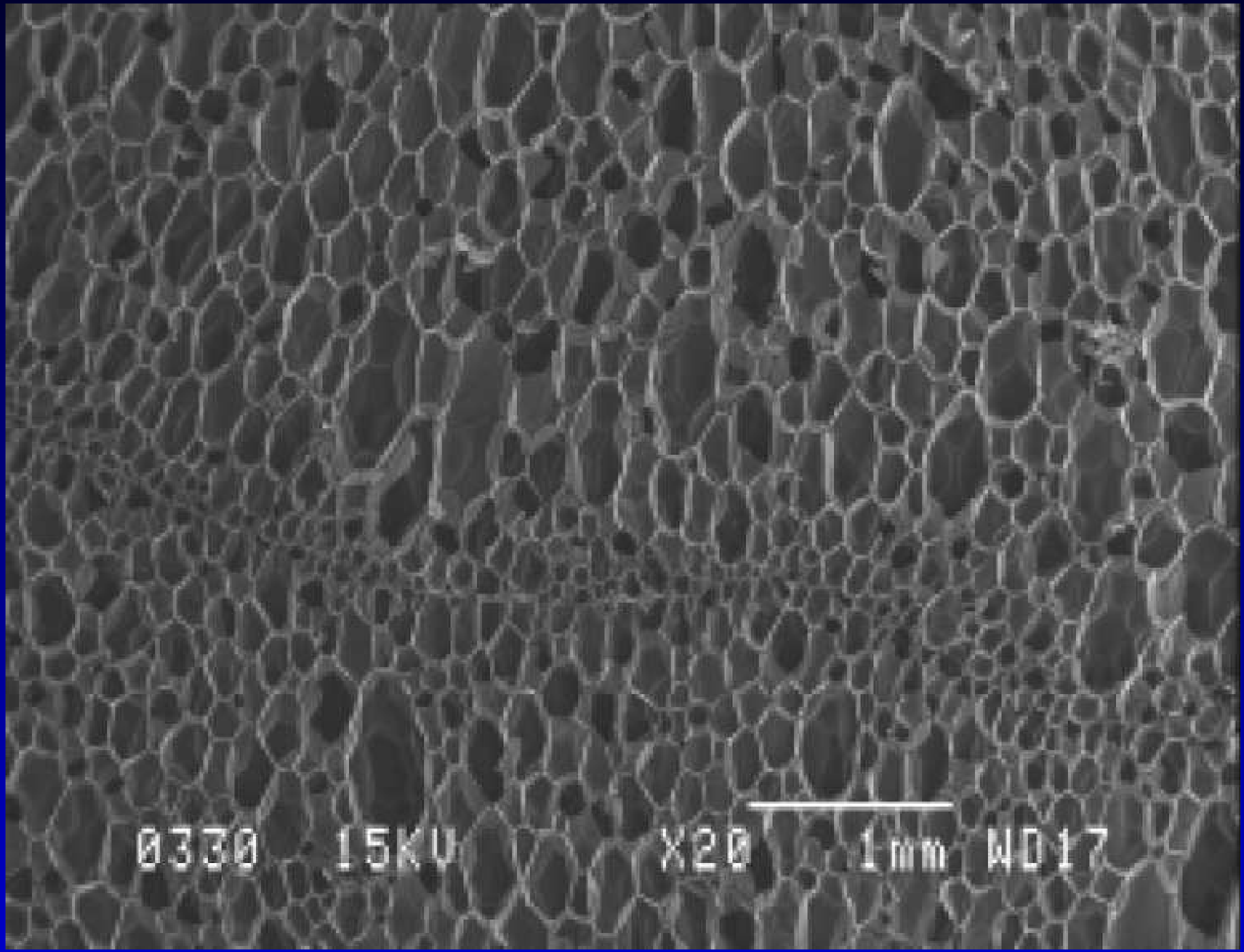
Voids in Foam

The foam on the space shuttle is sprayed on in layers (thus the acronym SOFI). Voids occur between layers.

Cured Layer

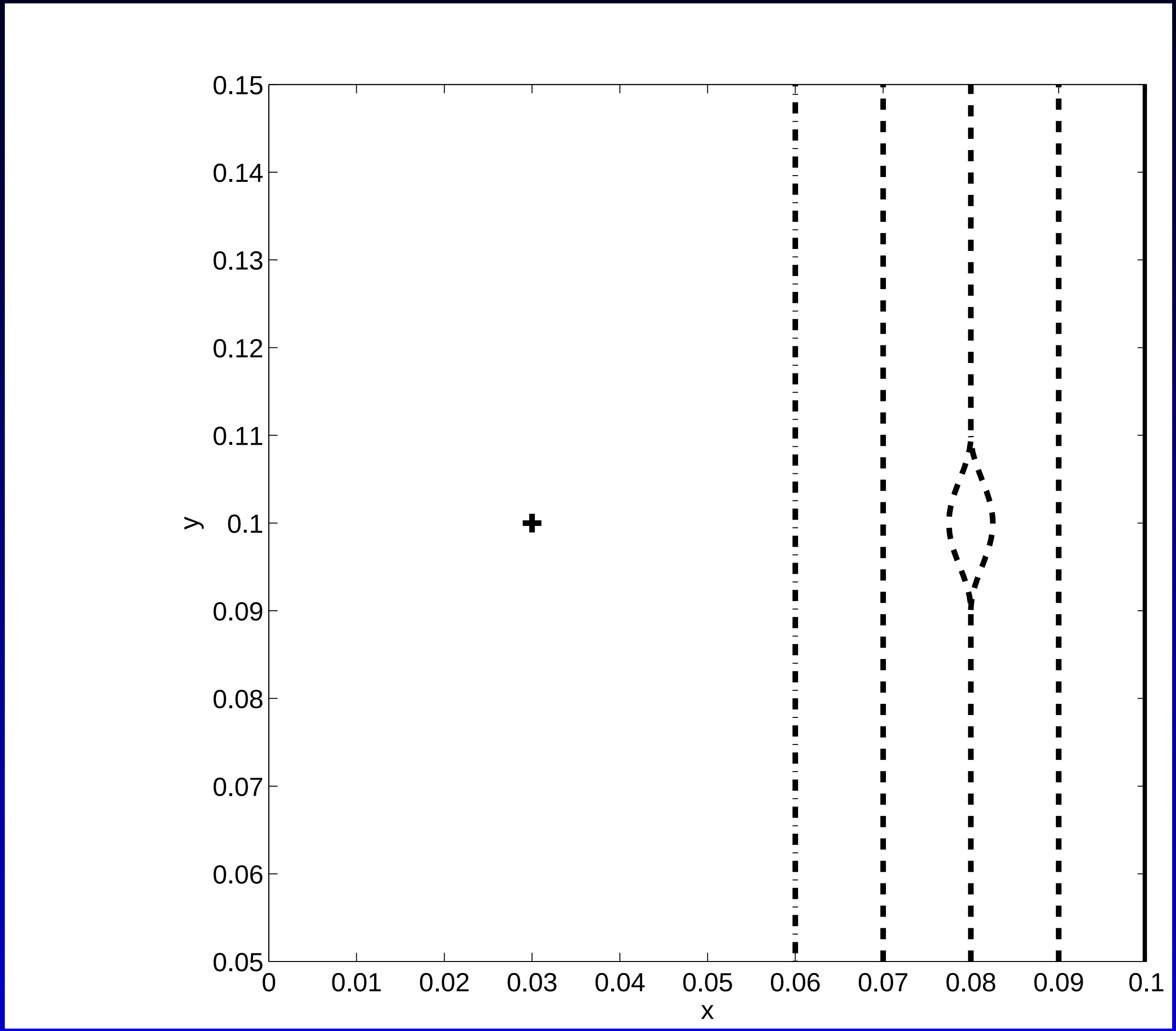
As the top of each layer cures, a thin knit line is formed which is of higher density (i.e., is comprised of smaller, more tightly packed polyurethane cells).

SOFI under 20X magnification



As the knit lines are on the order of 1 mm thick, they are generally ignored. But, what effect (if any) do the knit lines actually have on the interrogating signal? on detecting voids?

Sample Knit Lines with Void



Dashed lines represent knit lines, dot-dash is foam/air interface. Elliptical pocket (5 mm) between knit lines is a void. “+” marks the signal receiver. Back wall is perfect conductor.

2D Wave Equation

We assume the electric field to be polarized in the z direction, thus for $\vec{E} = (0, 0, E)$ and $\vec{x} = (x, y)$

$$\epsilon(\vec{x})\frac{\partial^2 E}{\partial t^2}(t, \vec{x}) - \nabla \cdot \left(\frac{1}{\mu(\vec{x})}\nabla E(t, \vec{x})\right) = -\frac{\partial J_s}{\partial t}(t, \vec{x})$$

where $\epsilon(\vec{x})$ and $\mu(\vec{x}) = \mu_0$ are the dielectric permittivity and permeability, respectively.

$$J_s(t, \vec{x}) = \delta(x)e^{-((t-t_0)/t_0)^4},$$

where $t_0 = t_f/4$ when t_f is the period of the interrogating pulse.

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Boundary Conditions

Consider $\Omega = [0, 0.1] \times [0, 0.2]$

- Reflecting (Dirichlet) boundary conditions (right)

$$[E]_{x=0.1} = 0$$

- First order absorbing boundary conditions (left)

$$\frac{\partial E}{\partial t} - \sqrt{\frac{1}{\epsilon(\vec{x})\mu_0}} \frac{\partial E}{\partial x}\bigg|_{x=0} = 0$$

- Symmetric boundary conditions (top and bottom)

$$\left[\frac{\partial E}{\partial y}\right]_{y=0, y=0.2} = 0$$

We use homogeneous initial conditions $E(0, \vec{x}) = 0$.

Modeling Knit Lines

- The speed of propagation in the domain is

$$c(\vec{x}) = \frac{c_0}{n(\vec{x})} = \sqrt{\frac{1}{\epsilon(\vec{x})\mu_0}},$$

  where c_0 is the speed in a vacuum and n is the index of refraction.

- We model knit lines by changing the index of refraction, thus effectively the speed in that region.

- Note that we currently ignore attenuation, focusing our attention on loss due to scattering from knit lines.

Clausius-Mossotti Equation

In order to relate the observed index of refraction of the entire foam, n_e, to the index in the low density region, n_1, we *may* apply the Clausius-Mossotti equation to get

$$\frac{n_e^2 - 1}{n_e^2 + 2} = 2(\nu\beta + 1 - \nu)\frac{n_1^2 - 1}{n_1^2 + 2},$$

where ν is the volume fraction of the foam occupied by the knit lines and $\beta = \frac{\rho_2}{\rho_1}$ represents the increase in density. Thus, if n_e is estimated via "time of flight" experiments, n_1 can be determined with reasonable values of ν and β.

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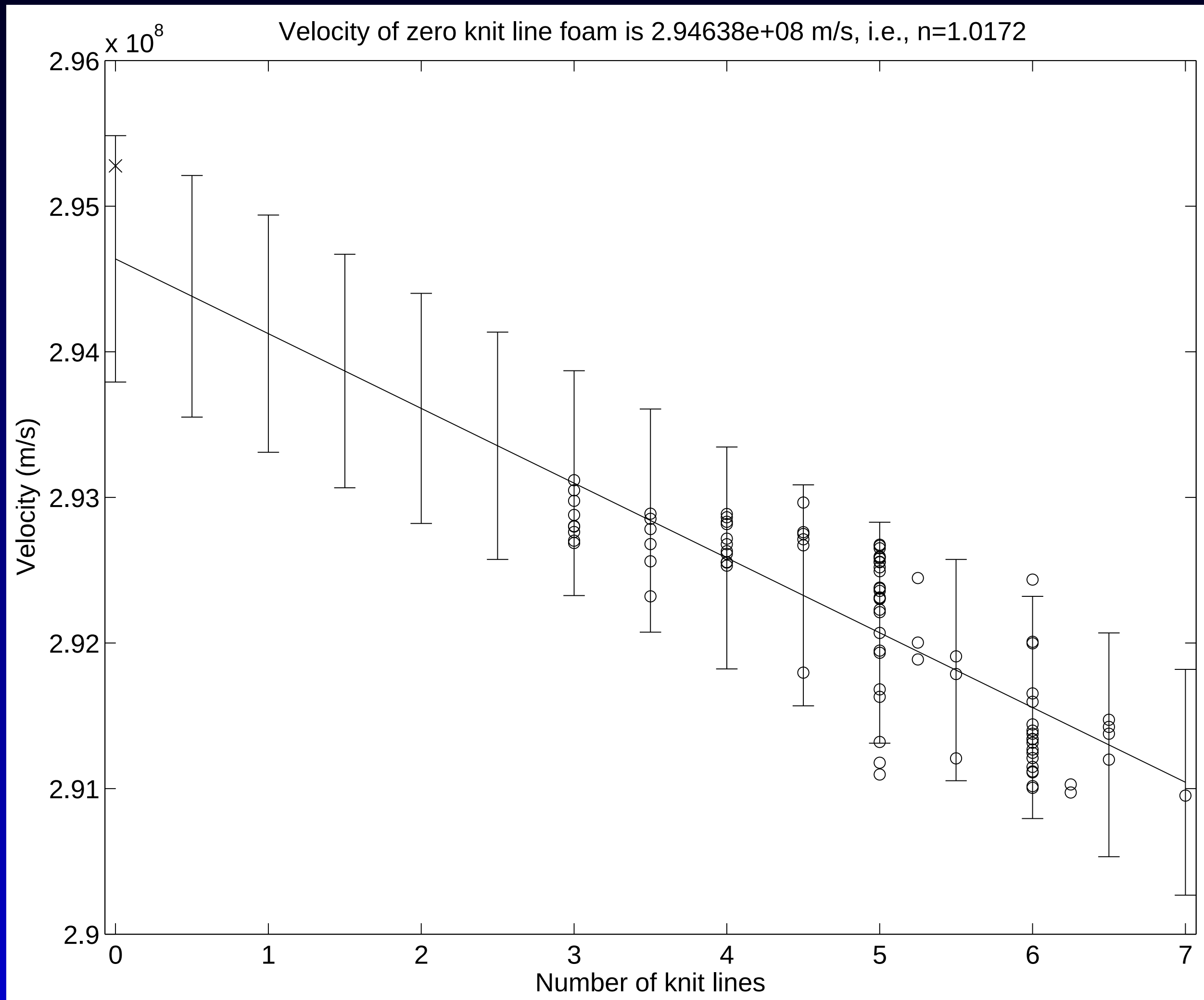
Experimental Data

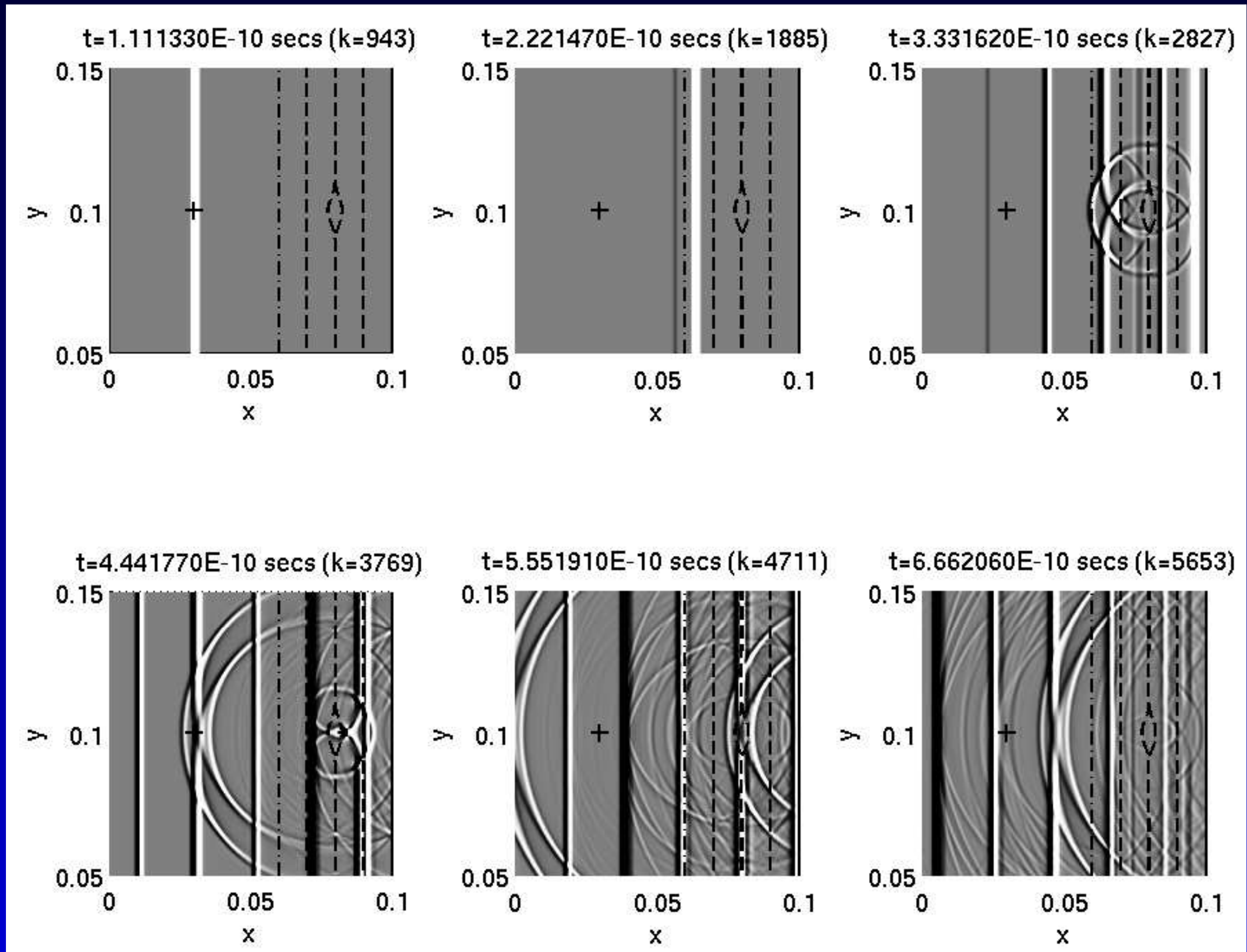


Applying linear regression we estimate that $n_1 = 1.0172$. (Clausius-Mossotti gives $n_1 = 1.0150$.) Using $n_e = (1 - \nu)n_1 + \nu n_2$, where $\nu = 0.05$ (.5mm knit line), we have the index in the knit line to be $n_2 = 1.1869$.

2D Numerical Discretization

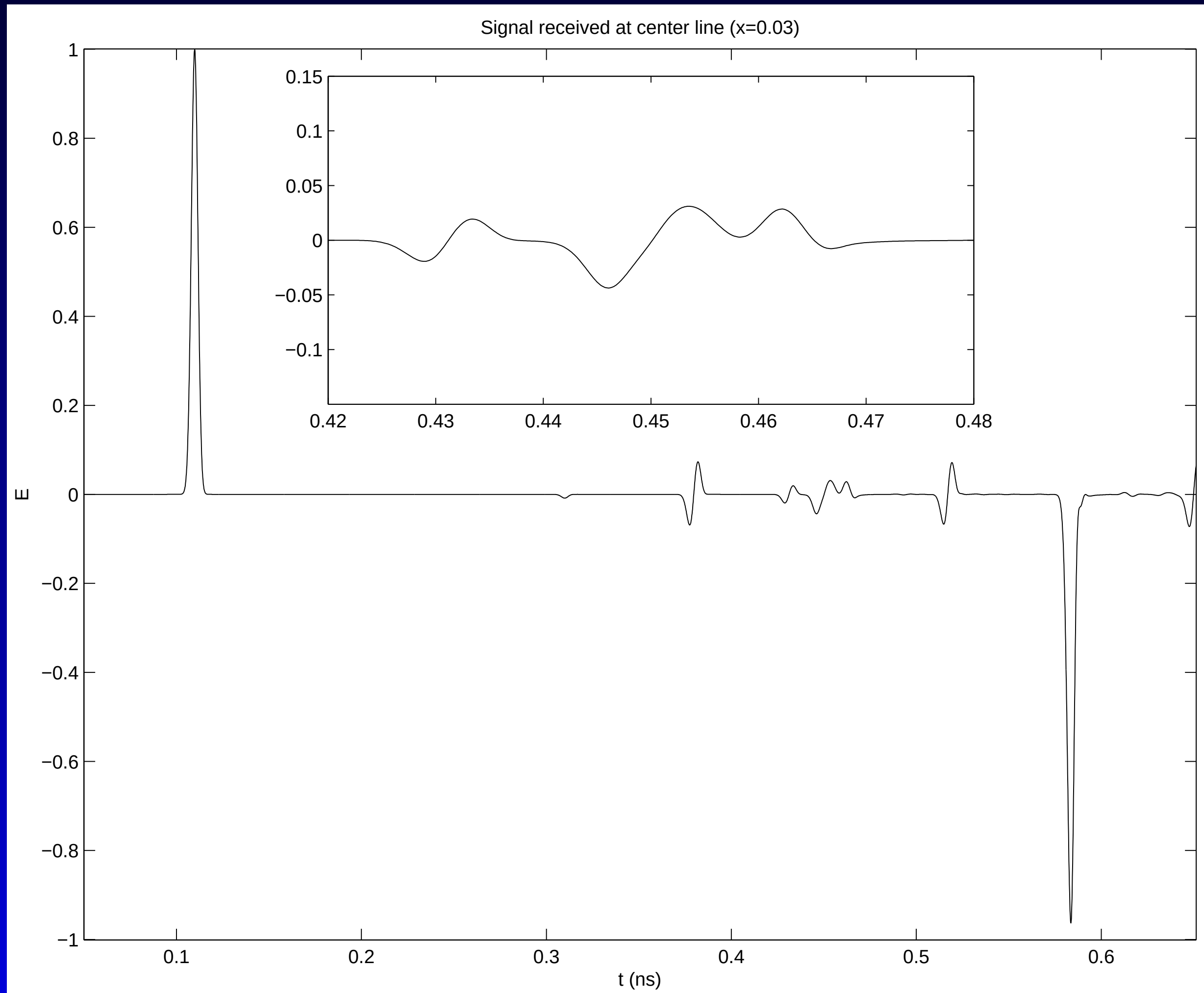
- Second order (piecewise linear) FEM in space
- Second order (centered) FD in time
- Linear solve (sparse)
  - Preconditioned conjugate-gradient
  - LU factorization
  - **Mass lumping (explicit)**
- Stair-stepping

Plane Wave Simulation



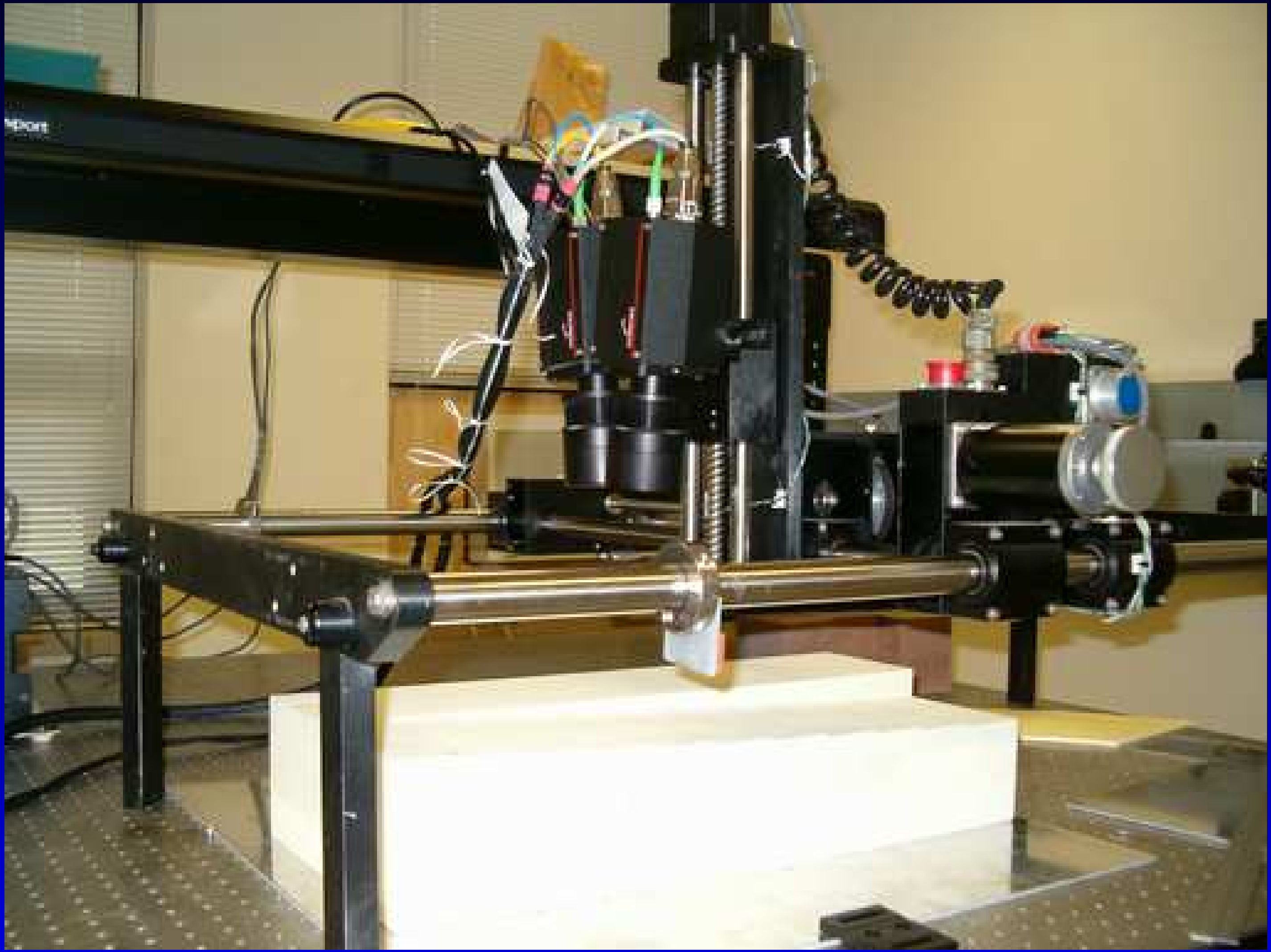
Source located at $x = 0$, receiver at $x = 0.03$.

Plane Wave Signal



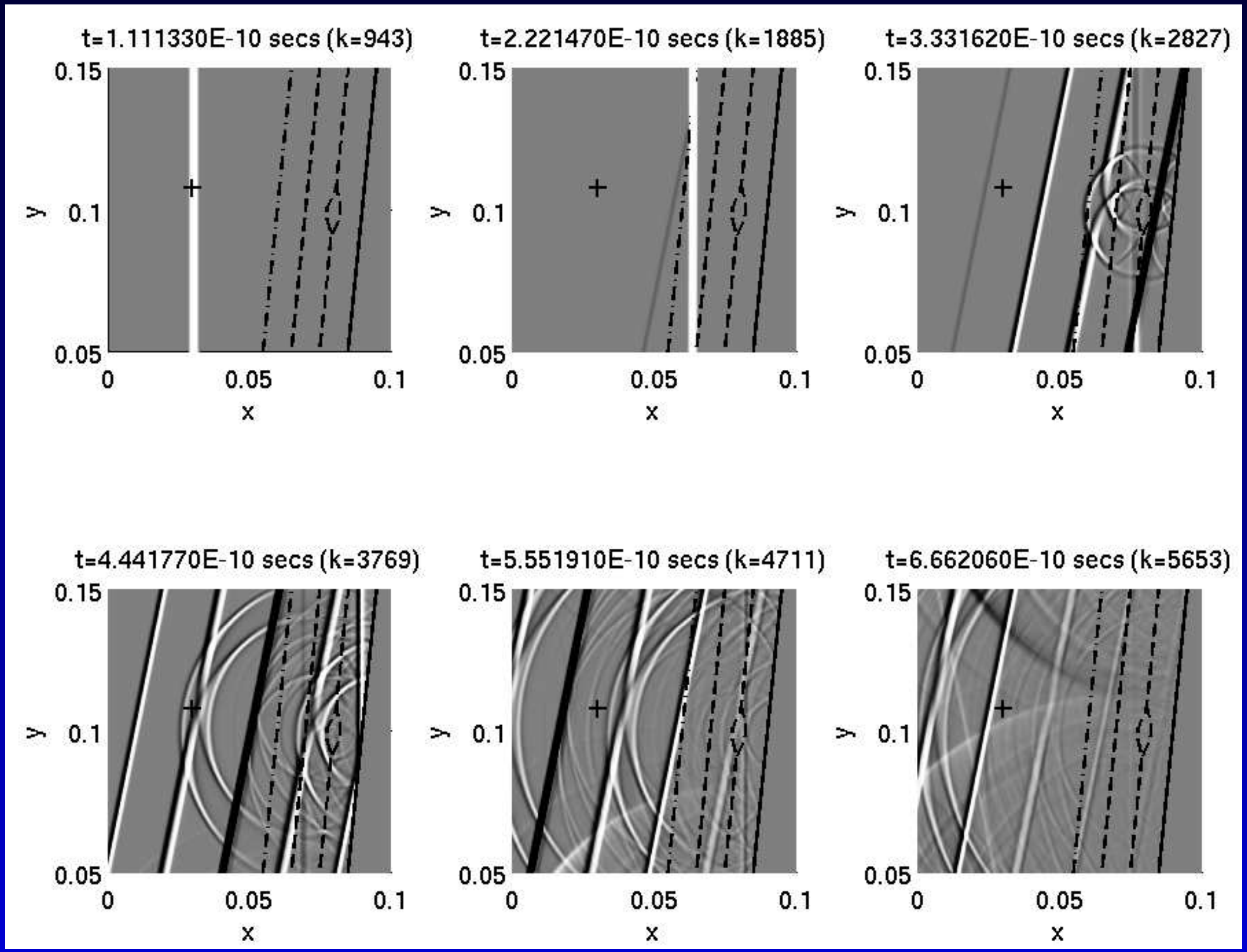
Interrogating signal simulates a sine curve truncated after one half period. Reflections off void are shown in inset.

Picometrix T-Ray Setup



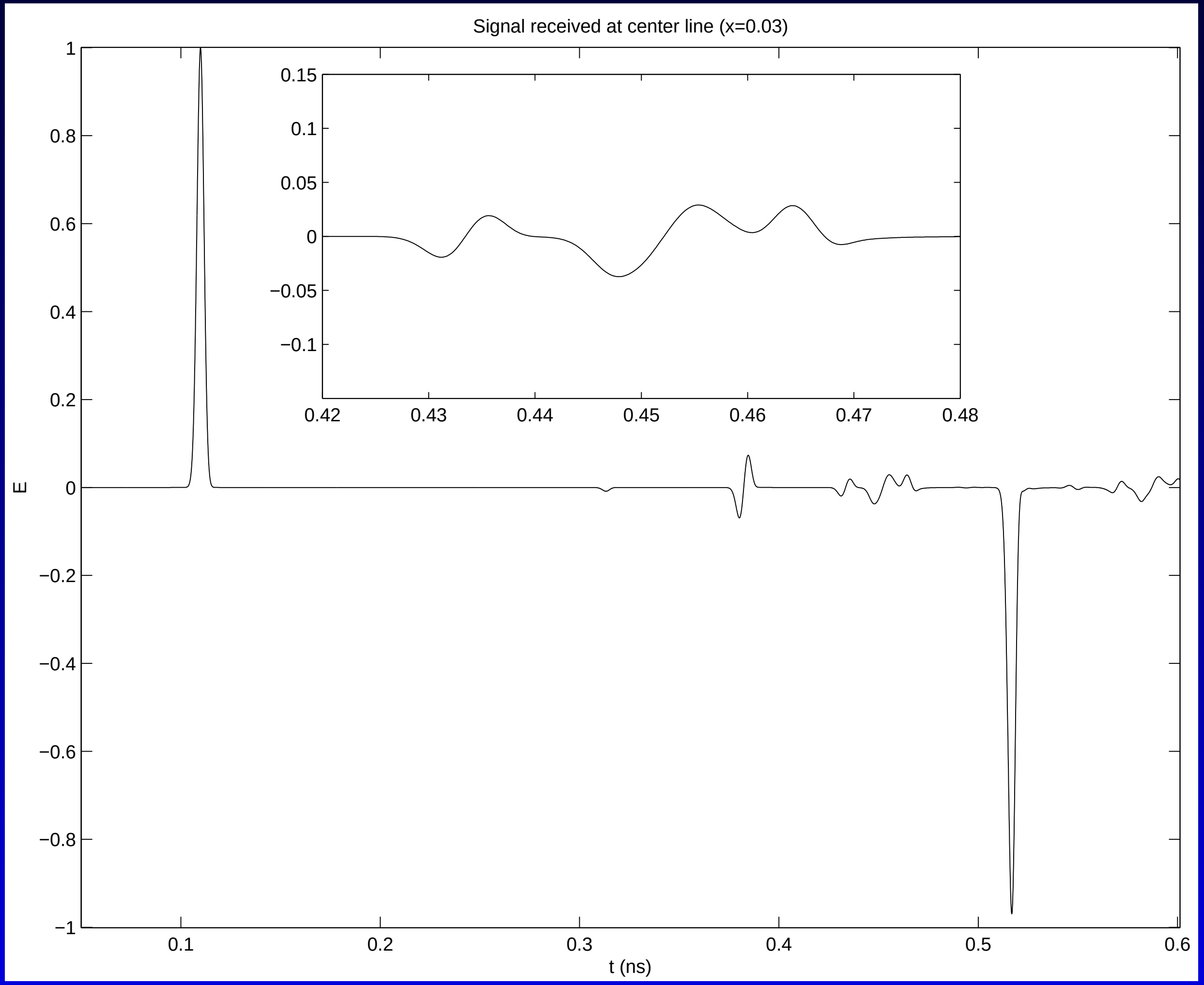
Note the non-normal incidence and ability to focus.

Oblique Plane Wave Simulation



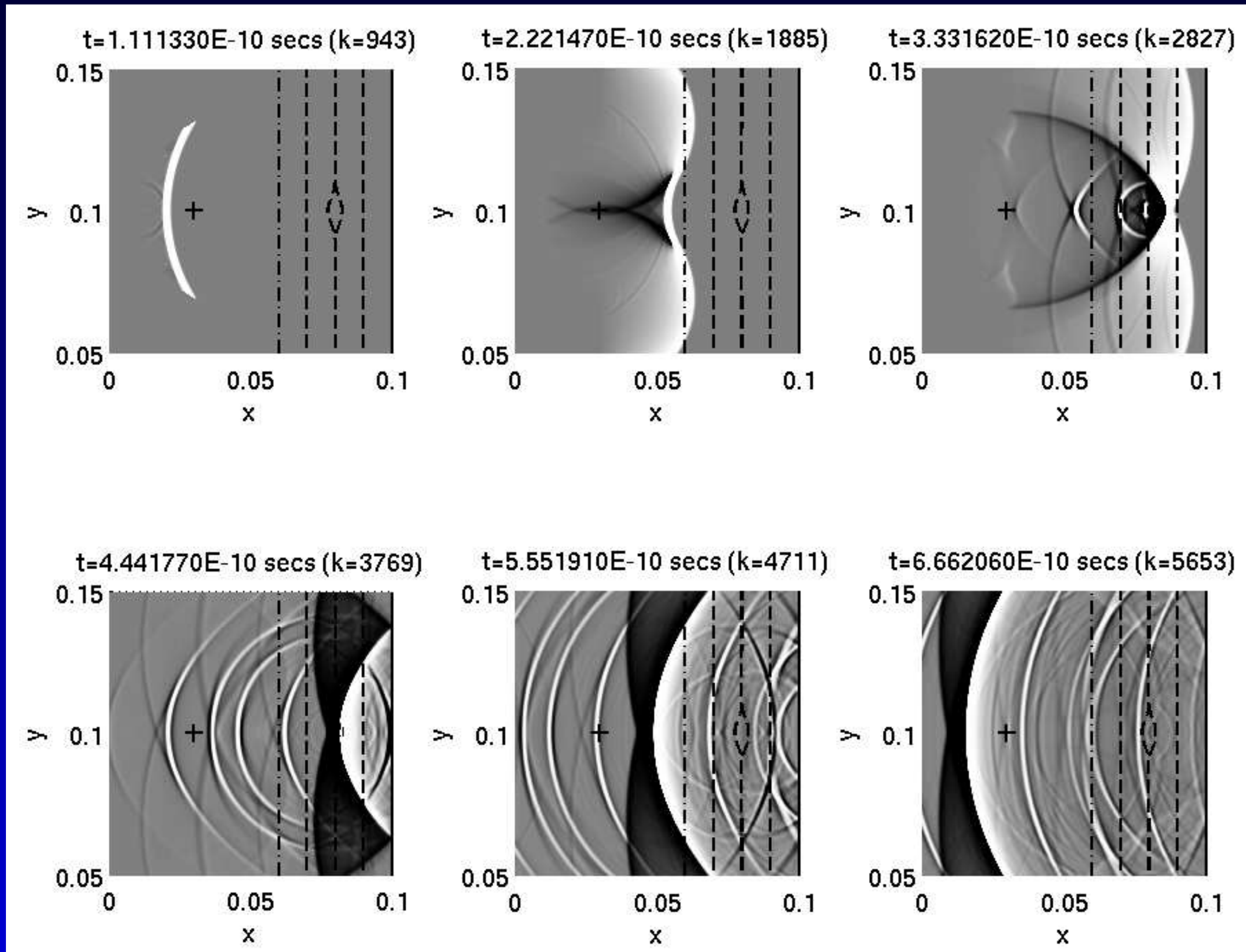
Source located at $x = 0$, receiver at $x = 0.03$, but raised to collect center of plane wave reflection.

Oblique Plane Wave Signal



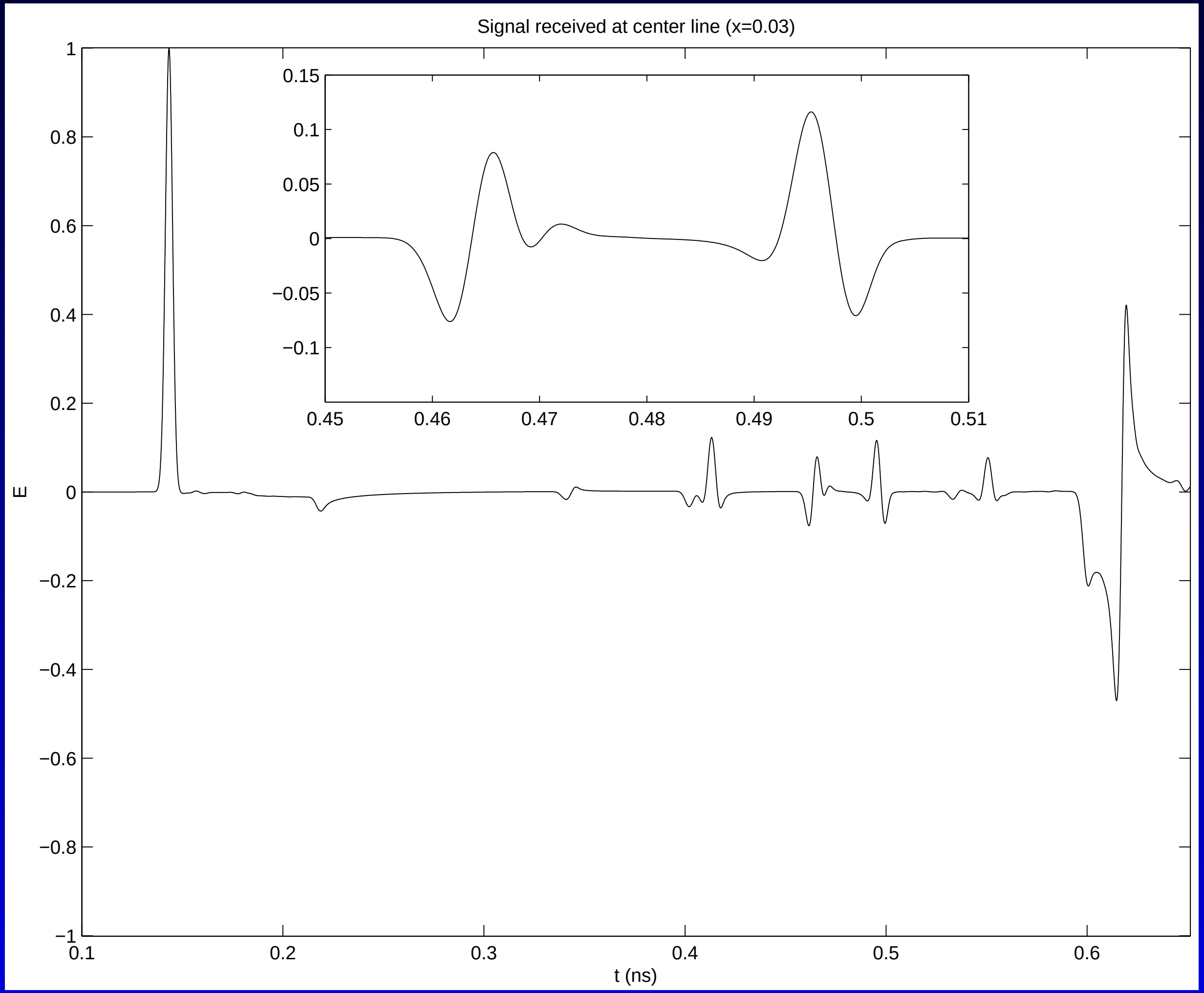
Nearly all of original signal returns even with an oblique angle of incidence. (Note: last knit line removed.)

Focused Wave Simulation



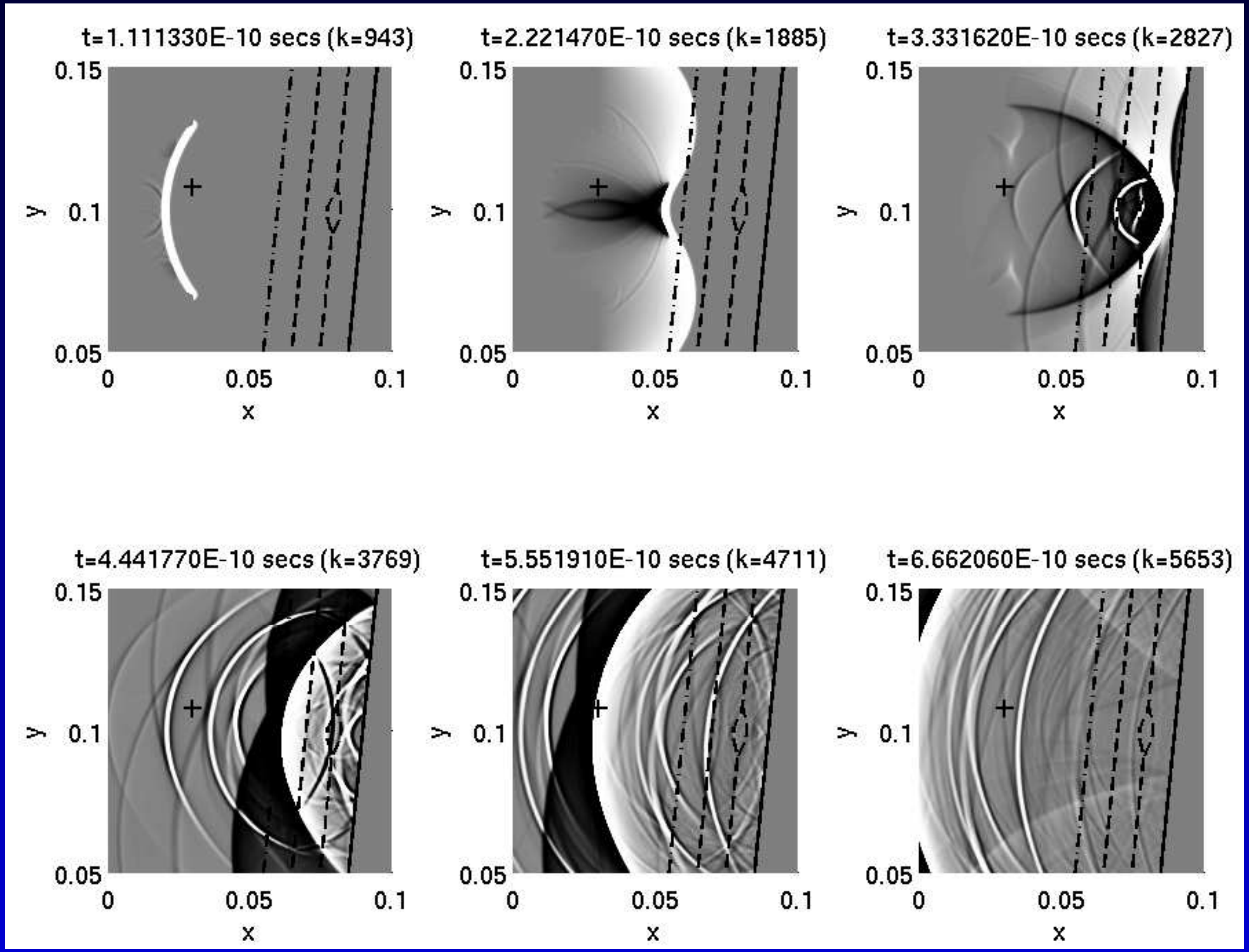
Source modeled using scattered field formulation of point source reflected from elliptical mirror. Receiver located at $x = 0.03$. Note top and bottom boundary conditions are now absorbing.

Focused Wave Signal



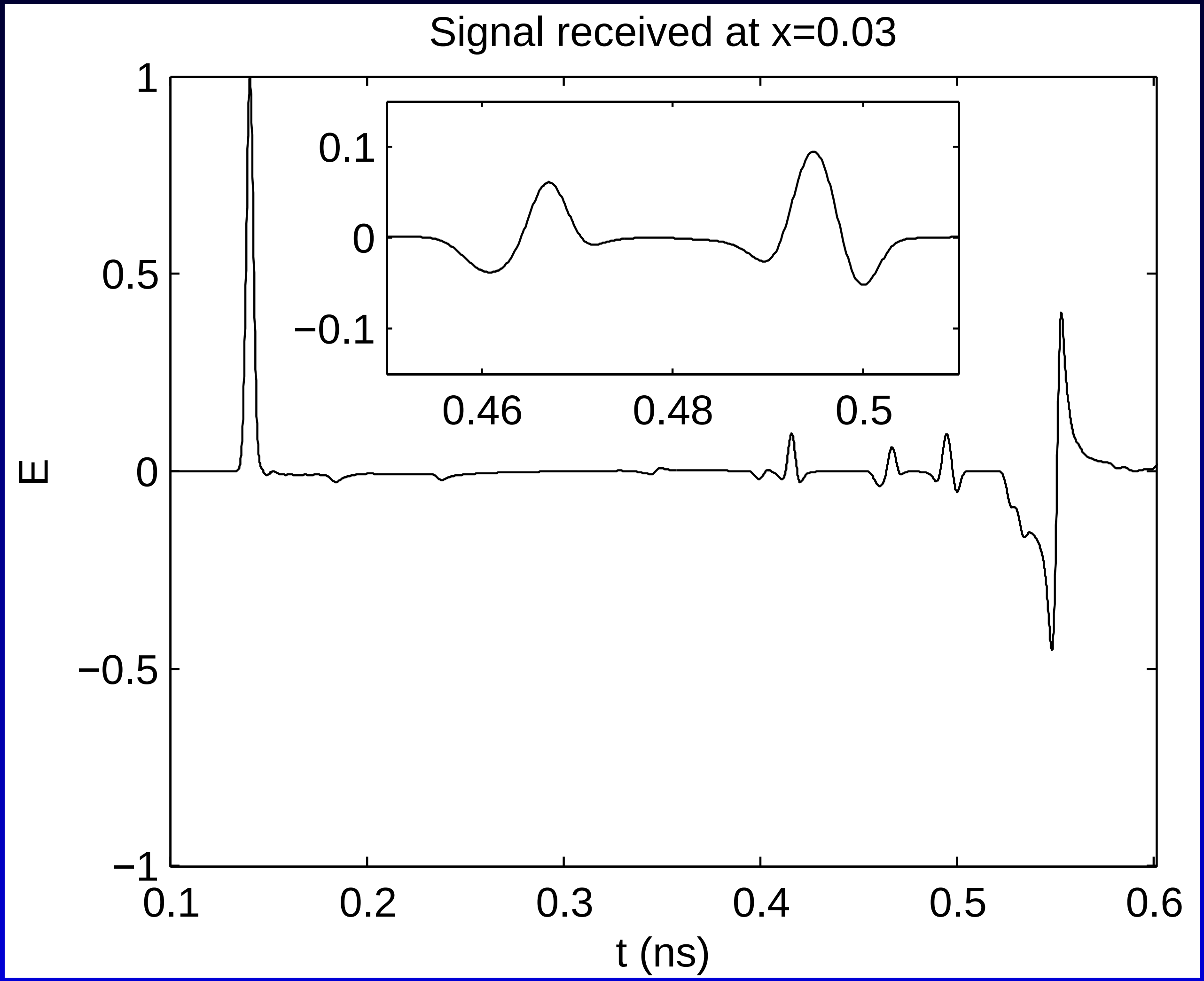
Although reflections off of void are larger, the total energy that returns is less than the plane wave simulation.

Oblique Focused Wave



Source modeled using scattered field formulation of point source reflected from elliptical mirror. Receiver located at $x = 0.03$, but raised to collect center of focused wave reflection.

Oblique Focused Wave Signal



Data received from non-normally incident, focused wave. Reflection from void is similar in magnitude to normal incidence focused wave.

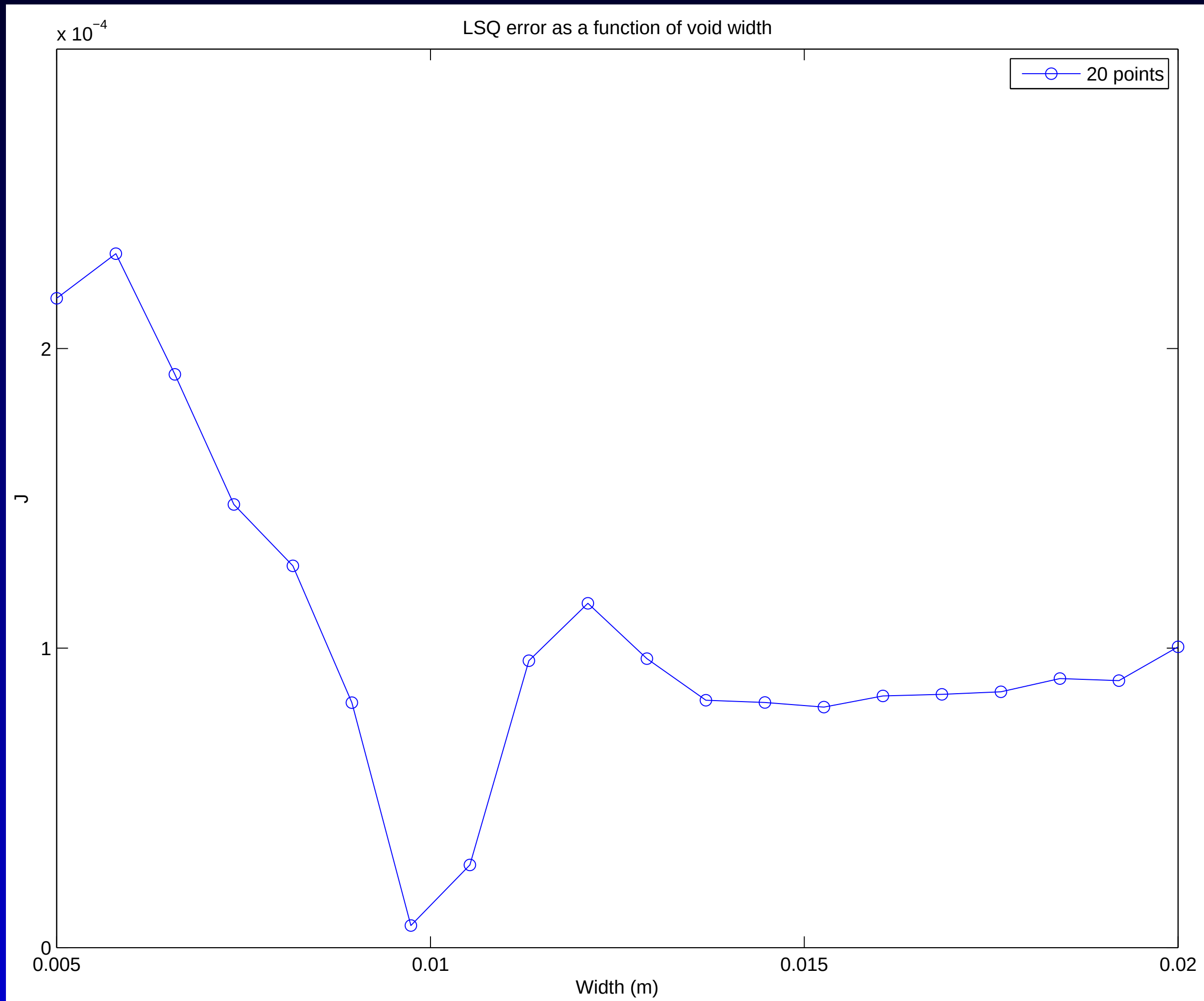
2D Void Inverse Problem

- Assume we have data, $\hat{E}_i$ at times t_i and $\mathbf{x} = \mathbf{x}^+$
- Given the width of an elliptical void w, we can simulate the electric field
- Estimate void width w by solving an inverse problem:

Find $w \in Q_{ad}$ such that the following objective function is minimized:

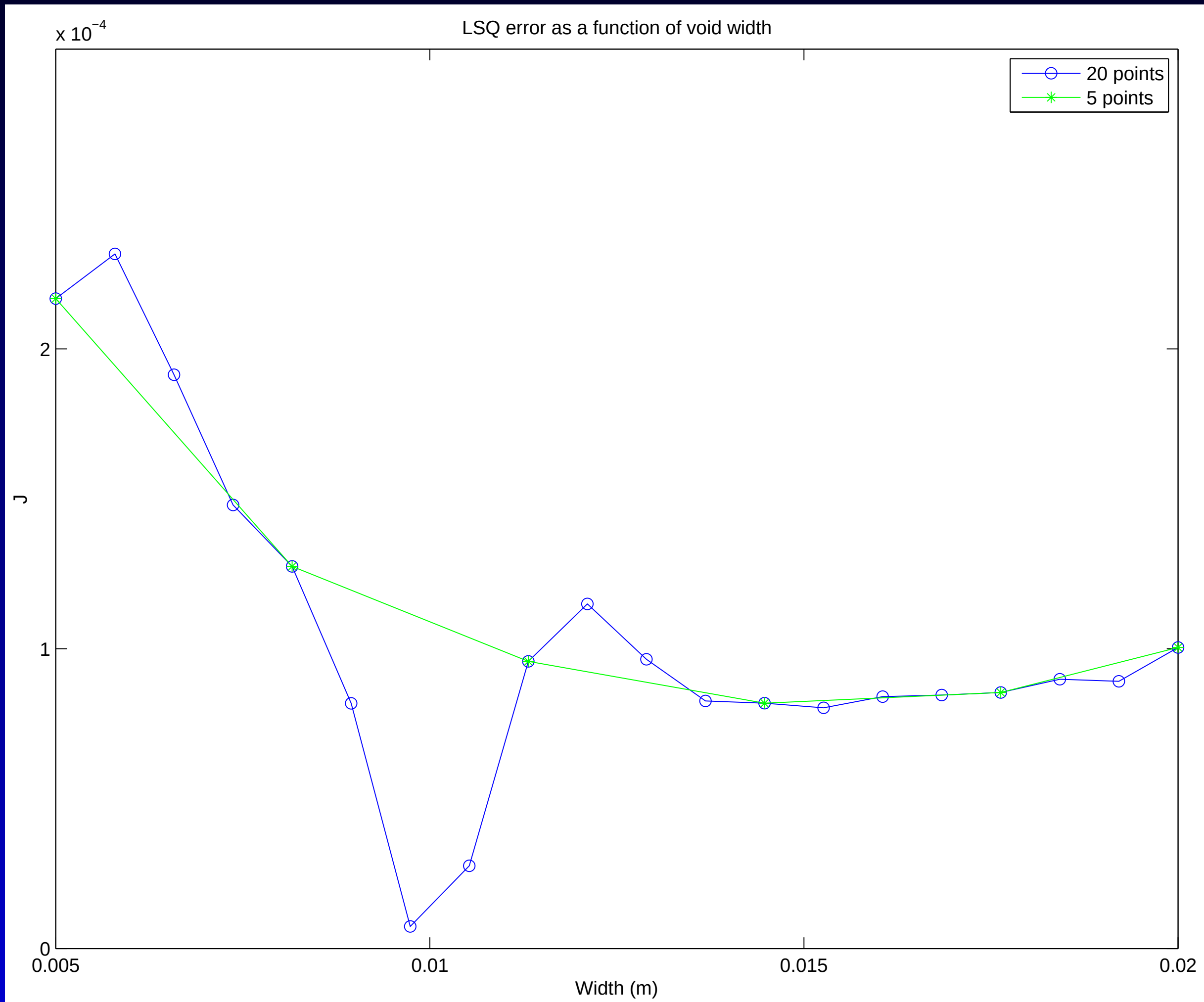
$$\mathcal{J}_1(w) = \frac{1}{2S} \sum_{i=1}^{S} |E(t_i, \mathbf{x}^+; w) - \hat{E}_i|^2.$$

$\mathcal{J}_1$ – Objective Function



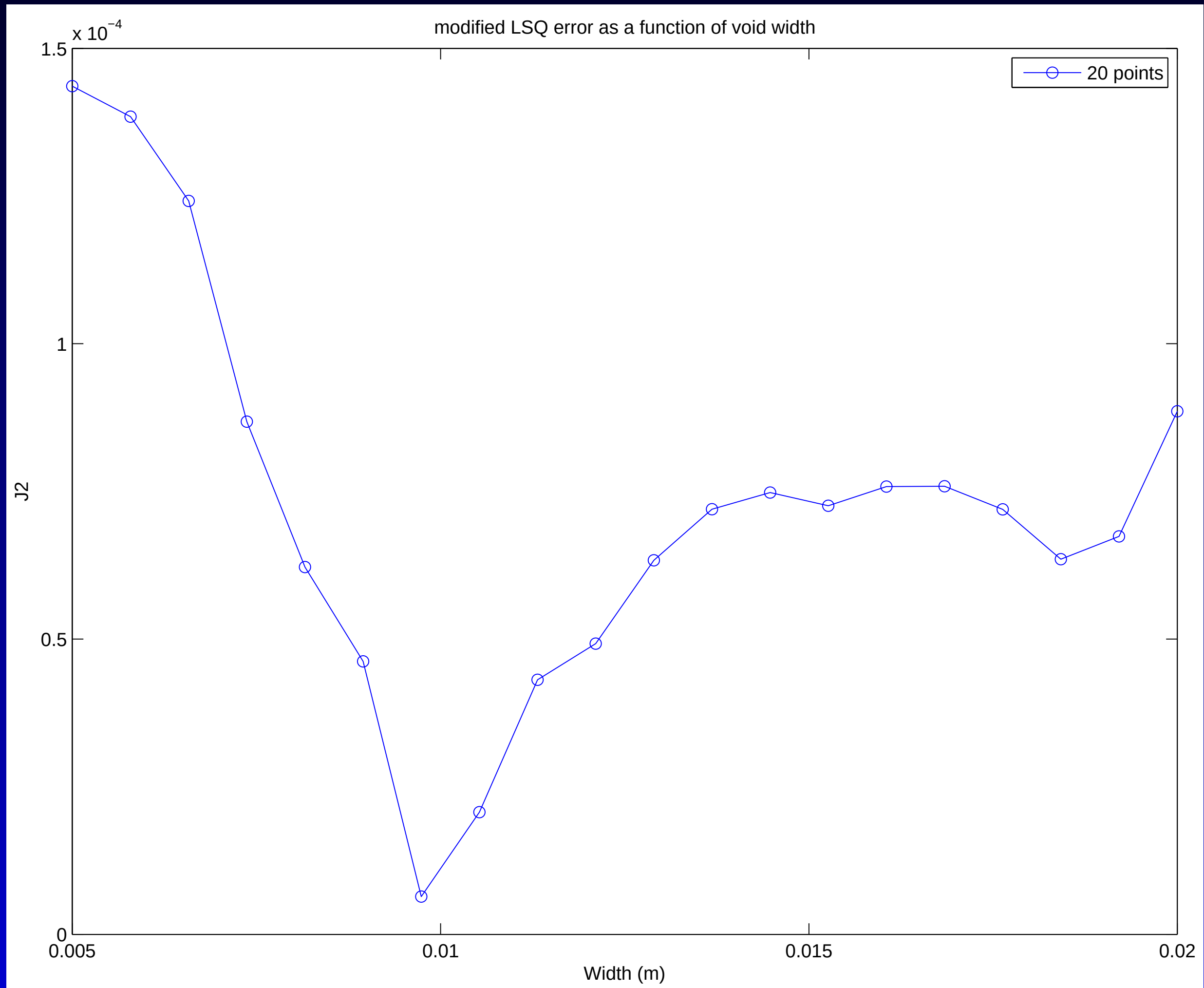
Location of the initial guess is crucial to minimizing with a gradient-based method.

$\mathcal{J}_1$ – Objective Function



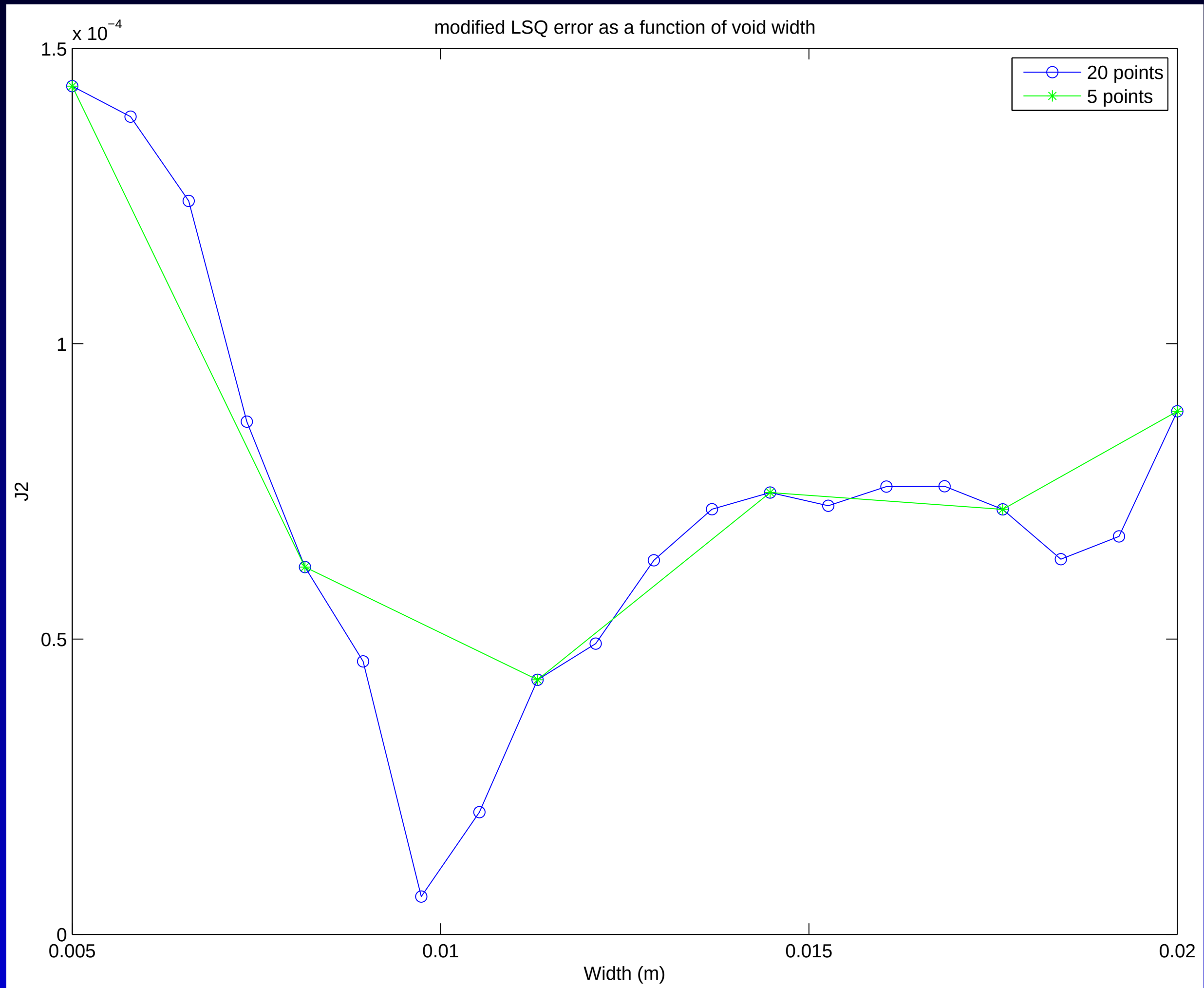
If we had only sampled the landscape with five points, we would have chosen poorly.

Improved Objective Function



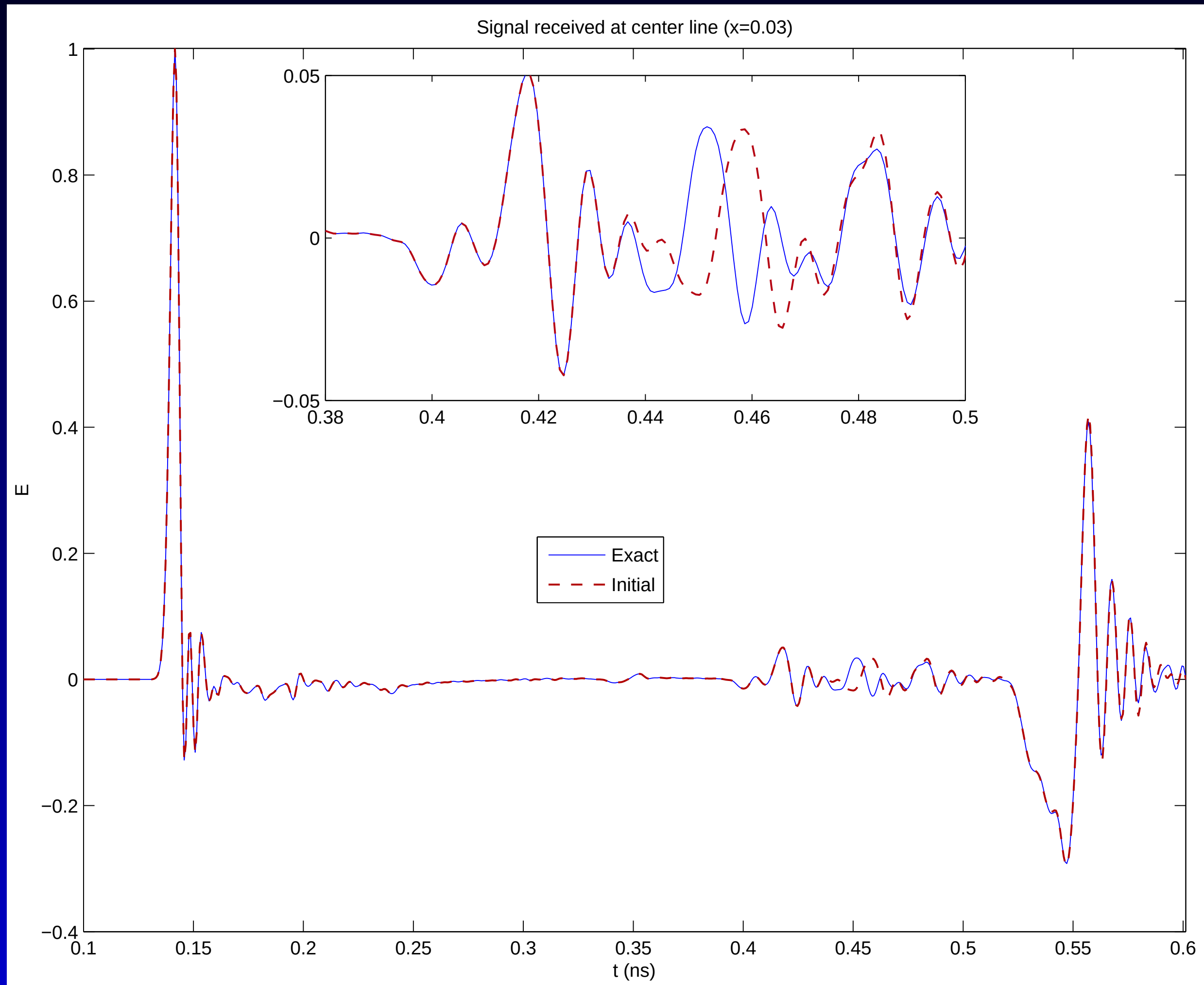
$$\mathcal{J}_2(w) = \frac{1}{2S} \sum_{i=1}^{S} \Big| |E(t_i, \mathbf{x}^+; w)| - |\hat{E}_i| \Big|^2 .$$

Improved Objective Function



Better representation of overall agreement of signals; more forgiving in chosing initial guess.

Inverse Problem Example



Low resolution example: using $w_0 = .008$, “optimal” value after 12 iterations is $w = .01$, exact value is $w = .01$.

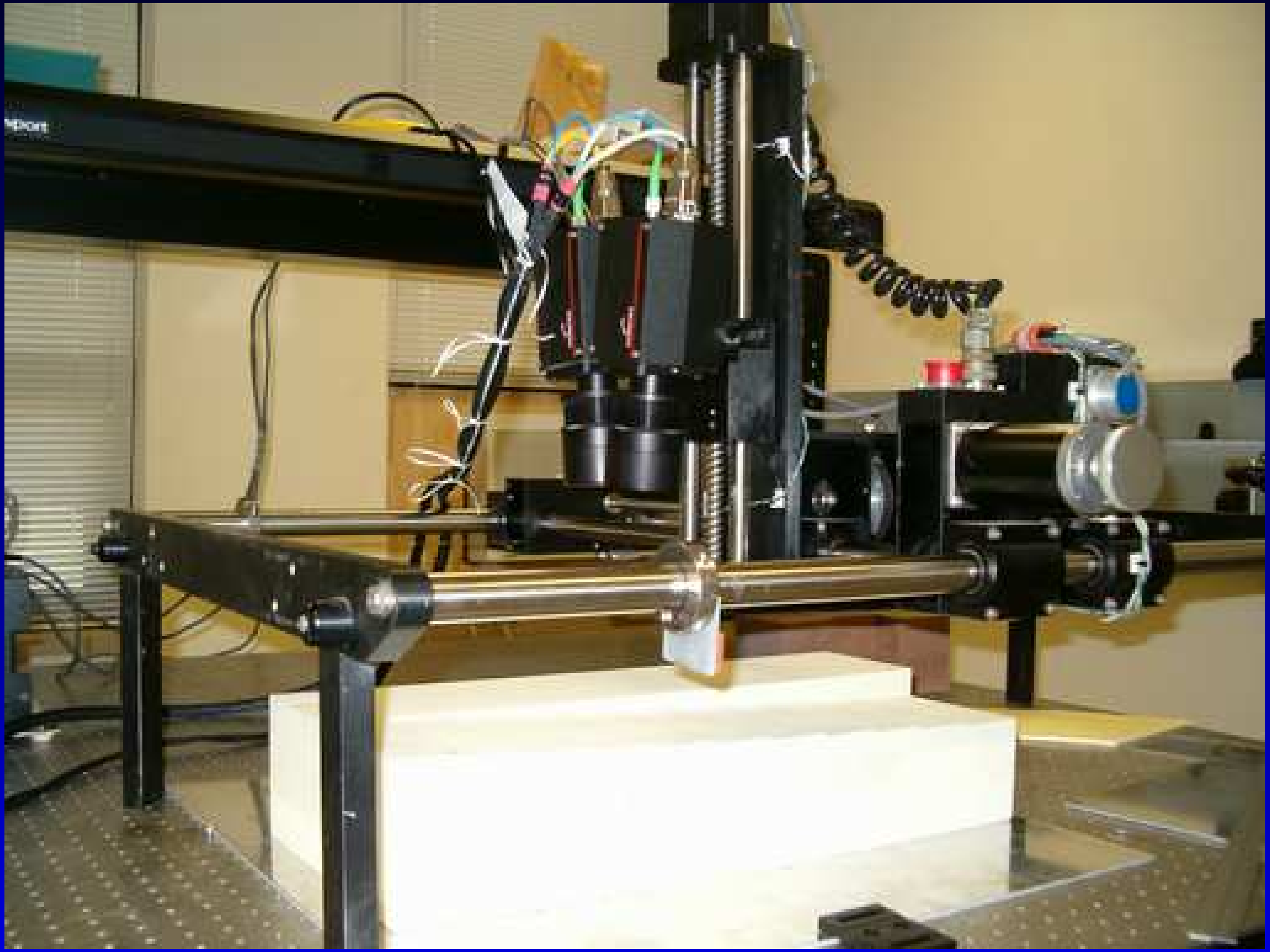
Concluding Remarks

With current power sources and detection devices, reflections from the front surface, voids, and knit lines are difficult to detect. Thus, for now, information is collected from the total reflection off the aluminum backing. More work needs to be done to match simulations to this data, including adding attenuation and using data to generate the simulated source.

Future Directions

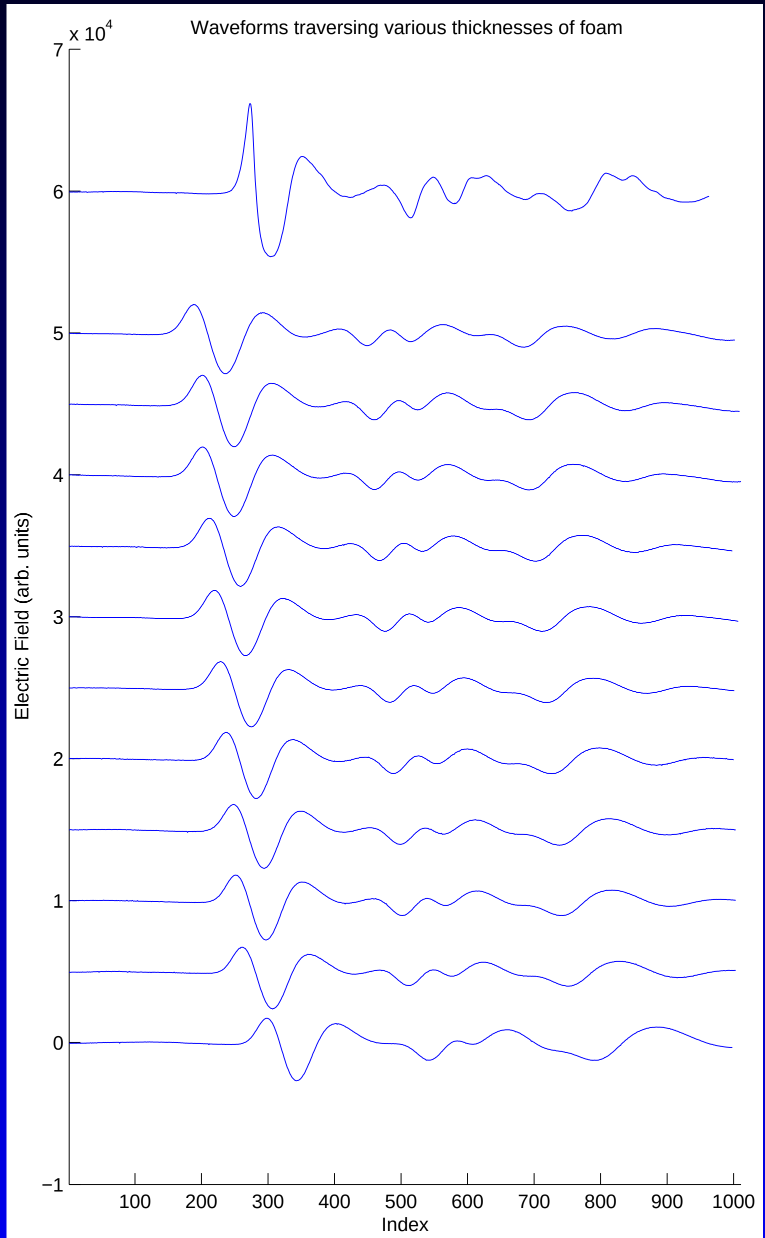
- Modeling Approaches
  - Scattering mechanism
  - Match attenuation observed in data
- Computational Methods
  - Higher order FE methods
  - No stair-stepping
  - Faster time-marching
  - ABC/PML

Picometrix T-Ray Setup



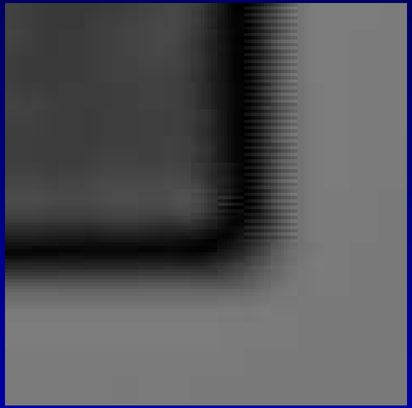
Step-block can be turned upside down to sample varying gap sizes.

THz through foam



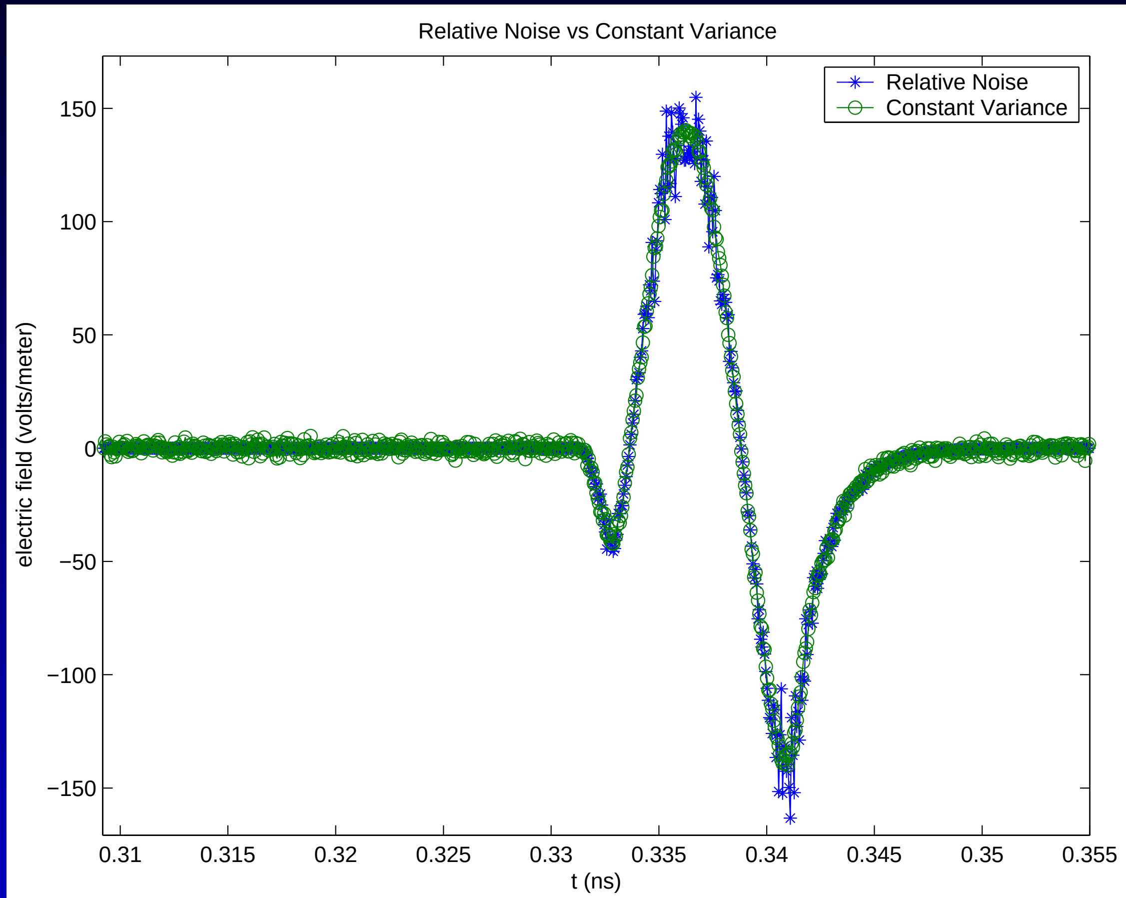
THz signal recorded after passing through foam of varying thickness, in a pitch-echo experiment.

Time-of-flight



Bitmap of time-of-flight recordings from step-block foam. Method clearly shows steep boundaries between foam and voids.

Relative vs Constant Variance Noise



The difference between data with relative noise added and data with constant variance noise added is clearly evident when E is close to zero or very large.