Finding damage in Space Shuttle foam

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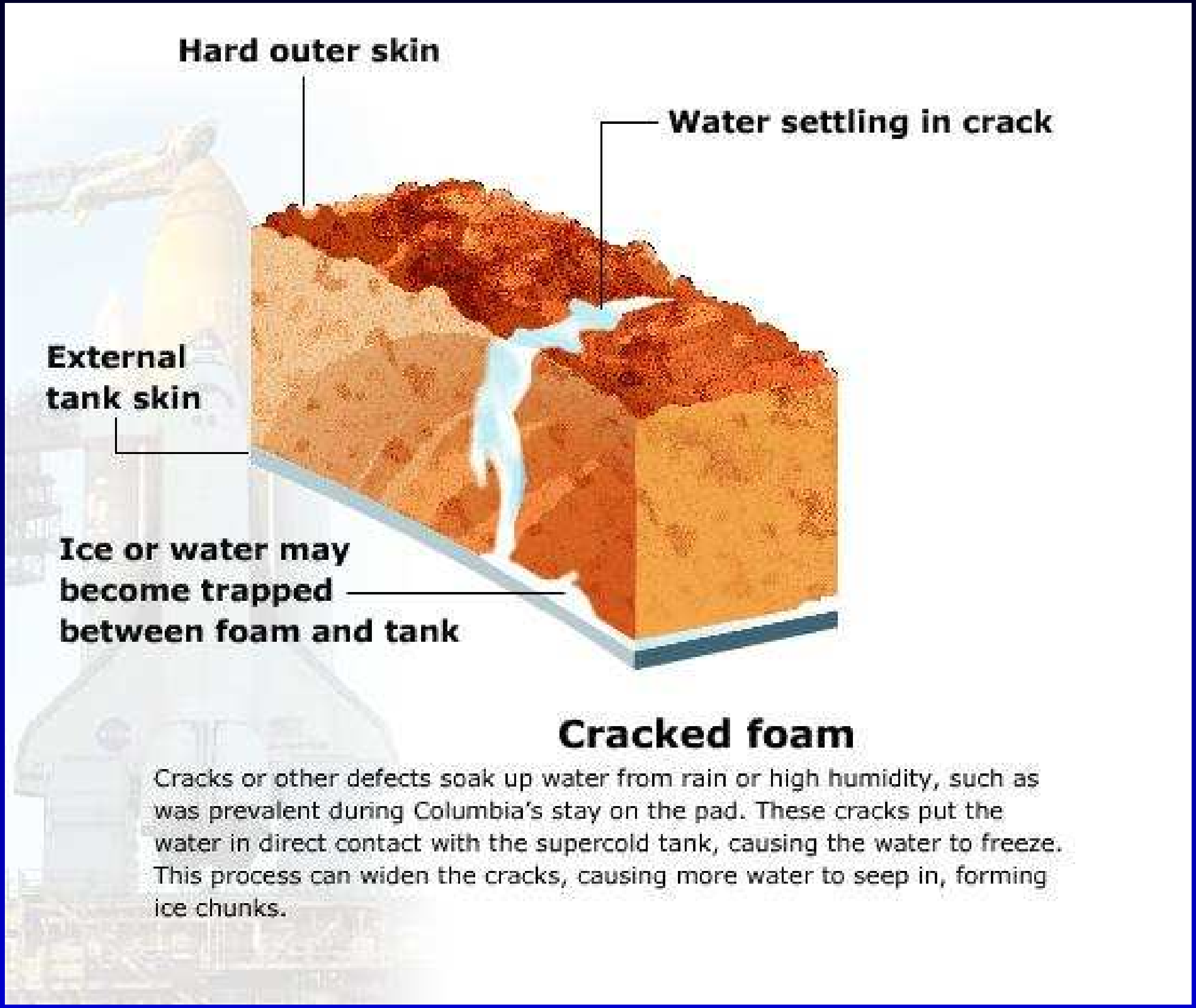
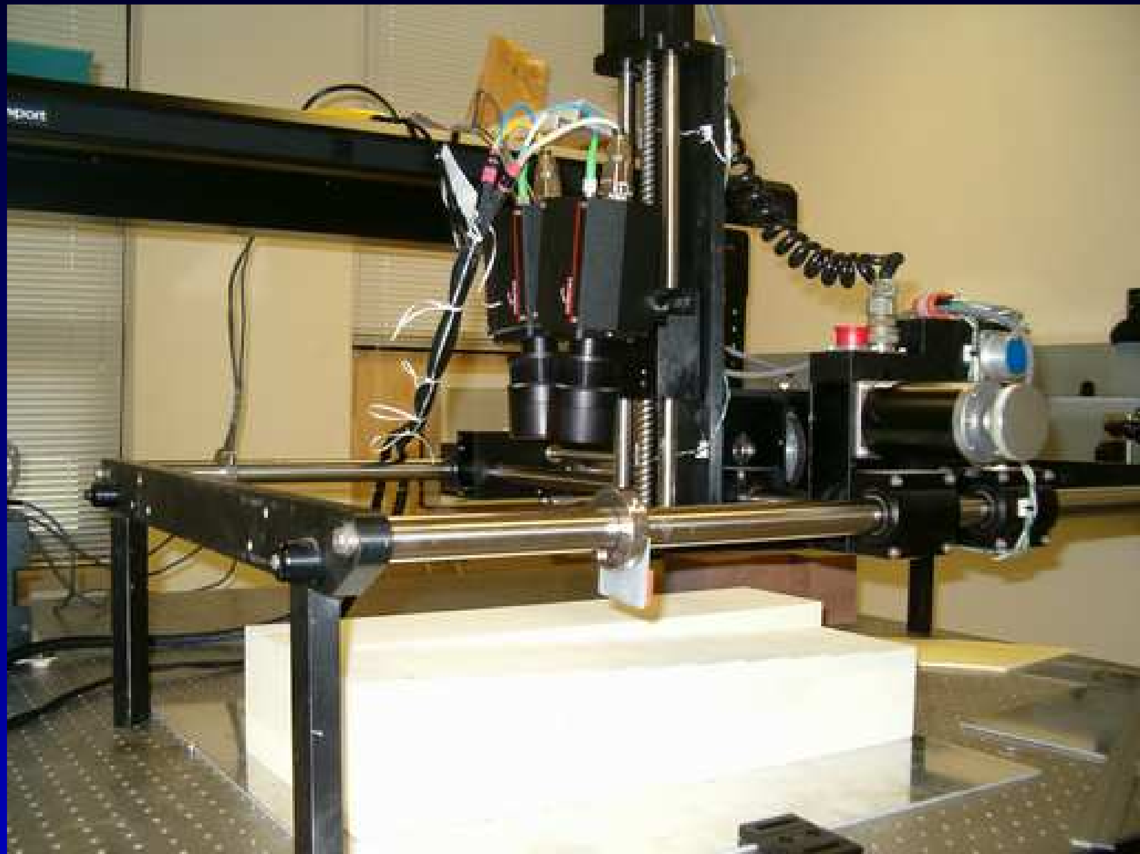
Motivating Application



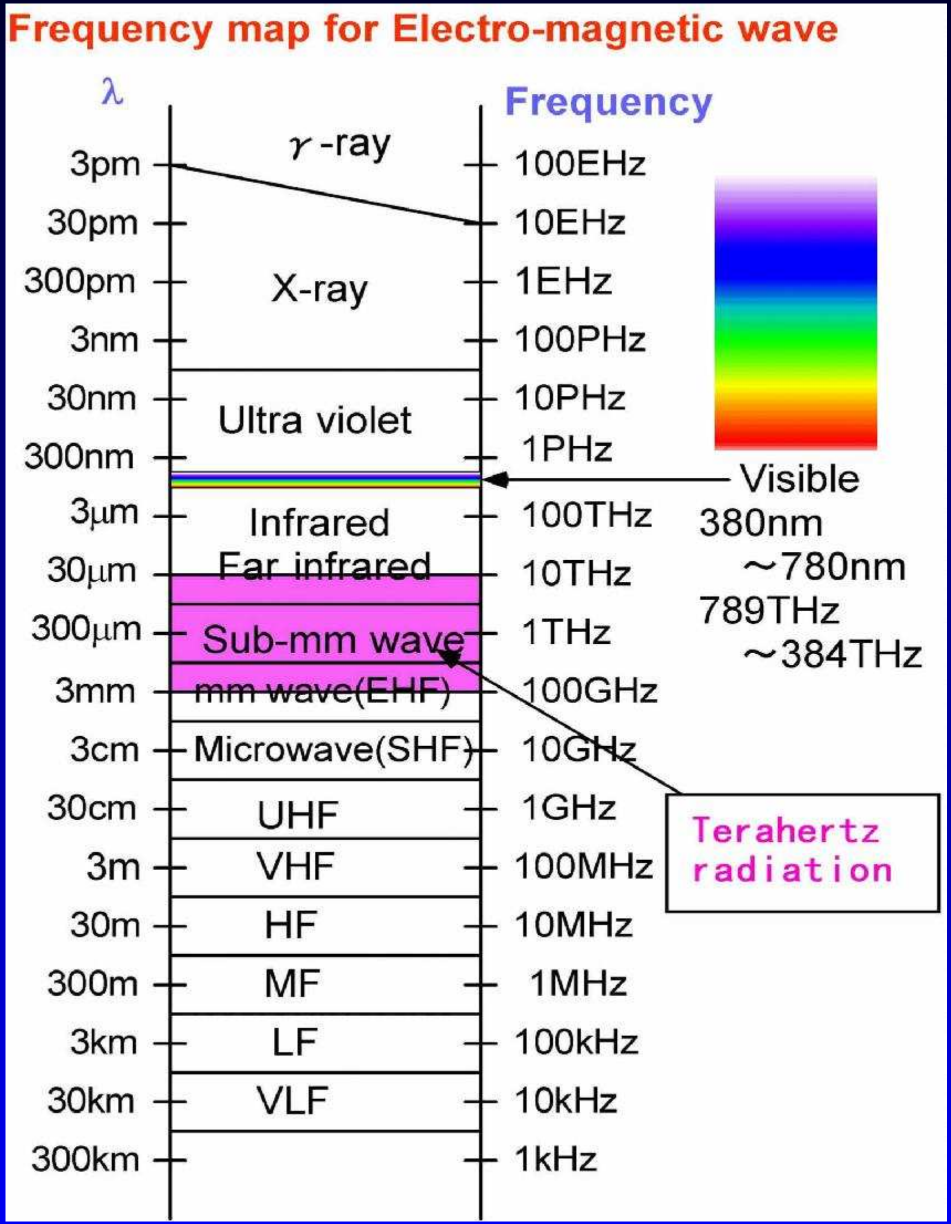
The particular motivation for this research is the detection of defects in the insulating foam on the space shuttle fuel tanks in order to help eliminate the separation of foam during shuttle ascent.

Picometrix T-Ray Setup



- Step-block can be turned upside down to sample varying gap sizes.
- Receiver and transmitter can be repositioned at various angles.
- Signal can be focused or collimated.

THz Gap


Frequency map for Electro-magnetic wave
λ
Frequency
γ-ray
X-ray
Ultra violet
Infrared
Far infrared
Sub-mm wave
mm wave(EHF)
Microwave(SHF)
UHF
VHF
HF
MF
LF
VLF
3pm
30pm
300pm
3nm
30nm
300nm
3μm
30μm
300μm
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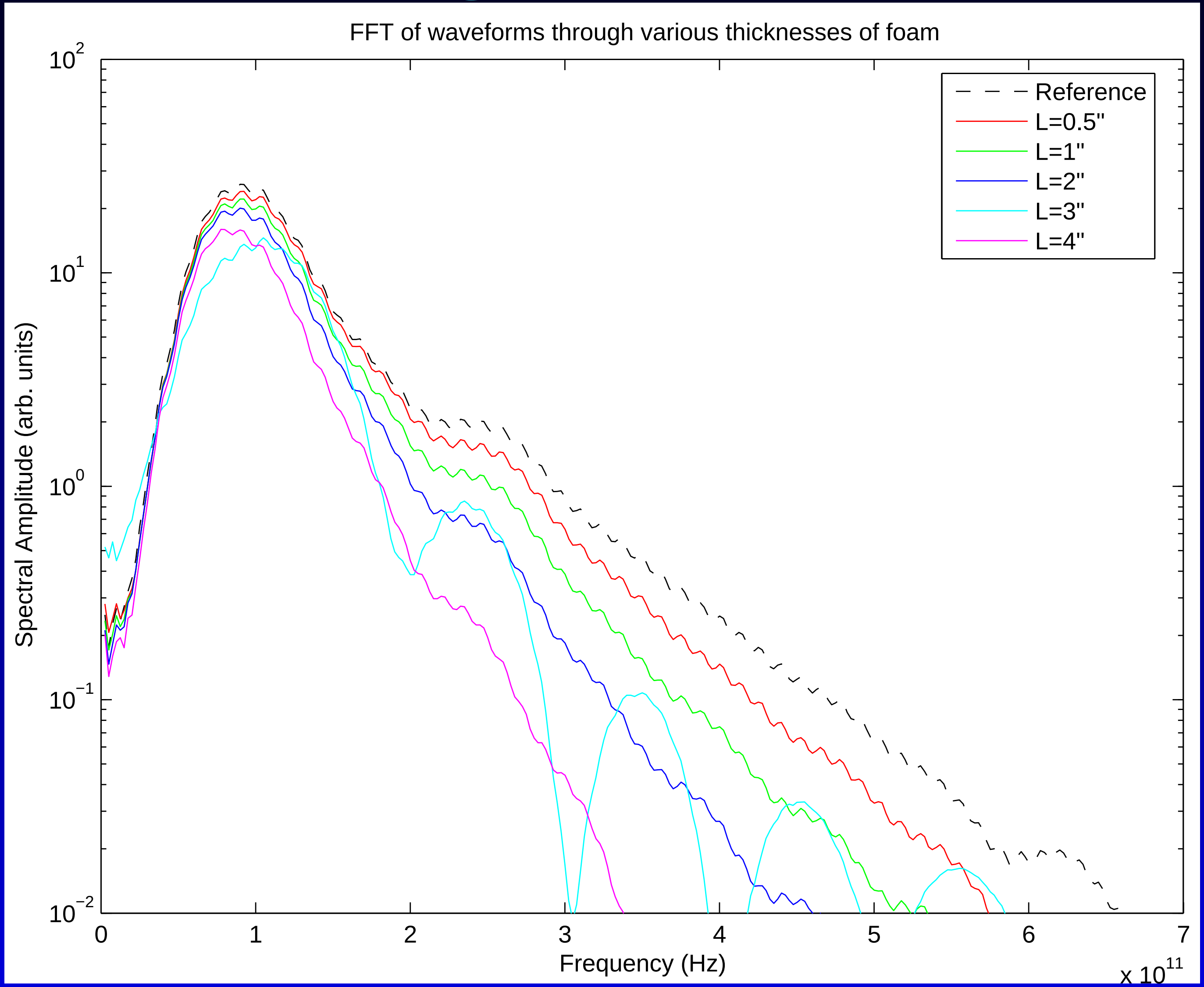
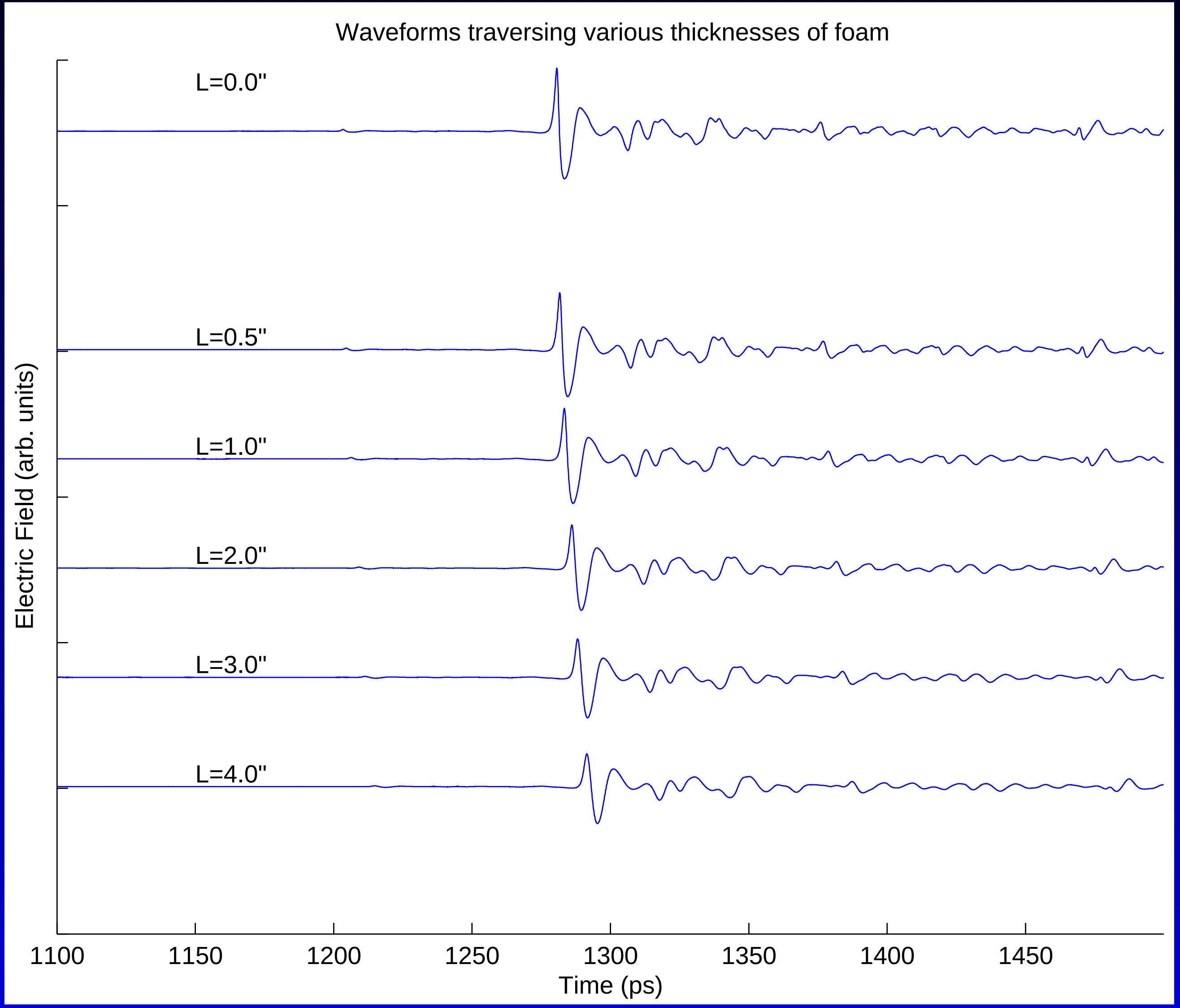
3km
30km
300km
100EHz
10EHz
1EHz
100PHz
10PHz
1PHz
100THz
10THz
1THz
100GHz
10GHz
1GHz
100MHz
10MHz
1MHz
100kHz
10kHz
1kHz
Visible
380nm
～780nm
789THz
～384THz
Terahertz
radiation

FFT of THz Signal



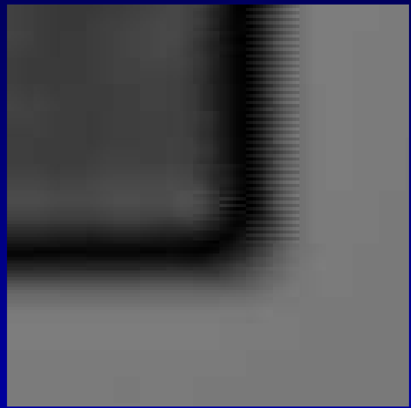
FFT of THz signal recorded after passing through foam of varying thickness, in a pitch-echo experiment.

THz Signal Through Foam



THz signal recorded after passing through foam of varying thickness, in a pitch-echo experiment.

Time-of-flight



Bitmap of time-of-flight recordings from step-block foam. Method clearly shows steep boundaries between foam and voids.

- Shows contrast, but does not accurately characterize damage.
- Less effective on horizontal discontinuities.

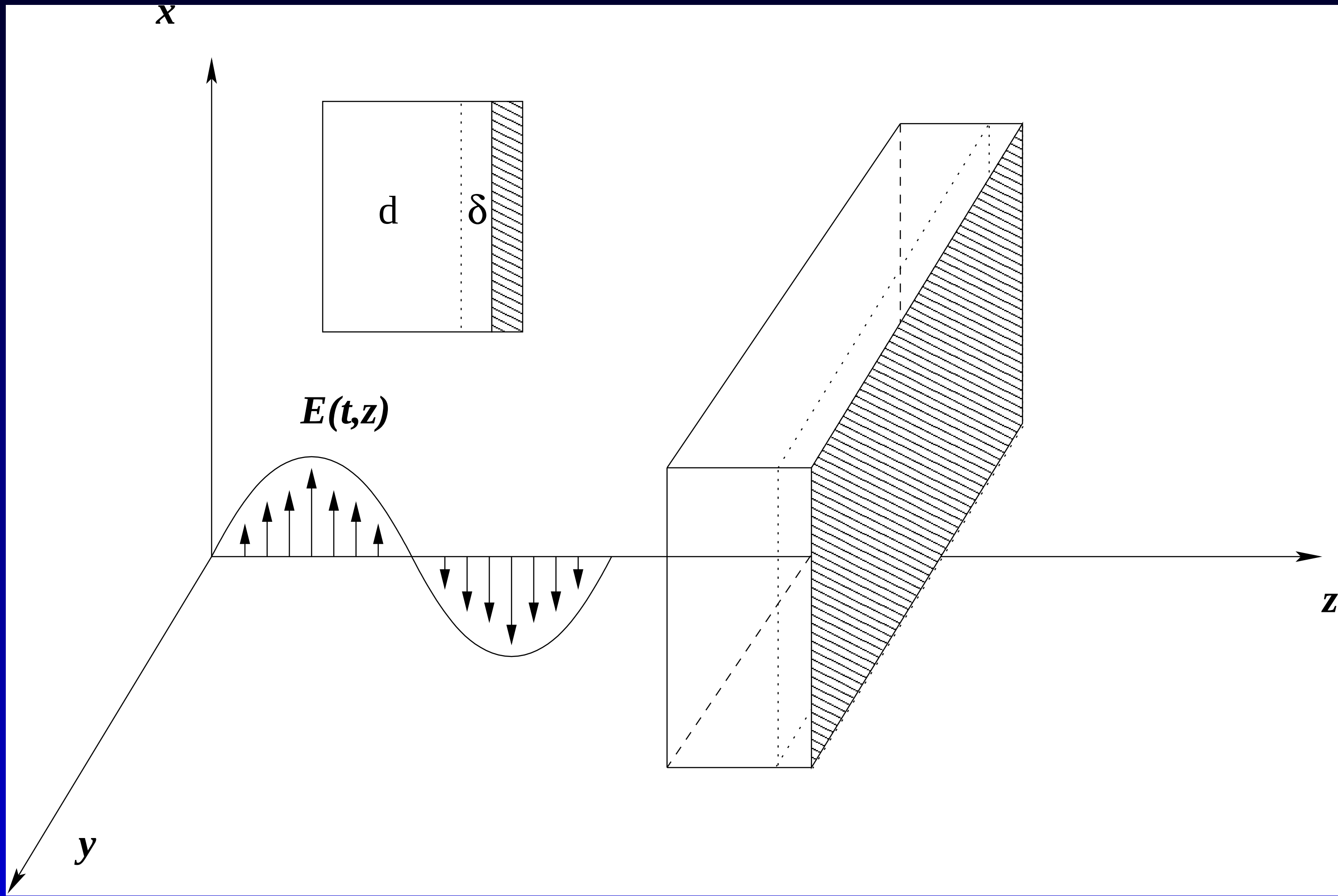
Outline

- 1D Gap Detection Inverse Problem
- 2D Void Detection
- Microstructure Modeling

Outline 1D Gap Detection

- Model
- Numerical Methods
- Inverse Problem
- Computational Results

Gap Detection Problem



Model

$$\mu_0\epsilon_0\epsilon_r\ddot{E} + \mu_0 I_\Omega \ddot{P} + \mu_0\sigma I_\Omega \dot{E} - E'' = -\mu_0 \dot{J}_s \qquad \text{in } \Omega \cup \Omega_0$$

$$\tau\dot{P} + P = \epsilon_0(\epsilon_s - \epsilon_\infty)E \qquad \text{in } \Omega$$

$$[\dot{E} - cE']_{z=0} = 0$$

$$[E]_{z=1} = 0$$

$$E(0,z) = \dot{E}(0,z) = 0$$

$$P(0,z) = 0$$

where

$$J_s(t,z) = \delta(z) sin(\omega t) I_{[0,t_f]}(t)$$

and

$$\epsilon_r = (1 + (\epsilon_\infty - 1)I_\Omega).$$

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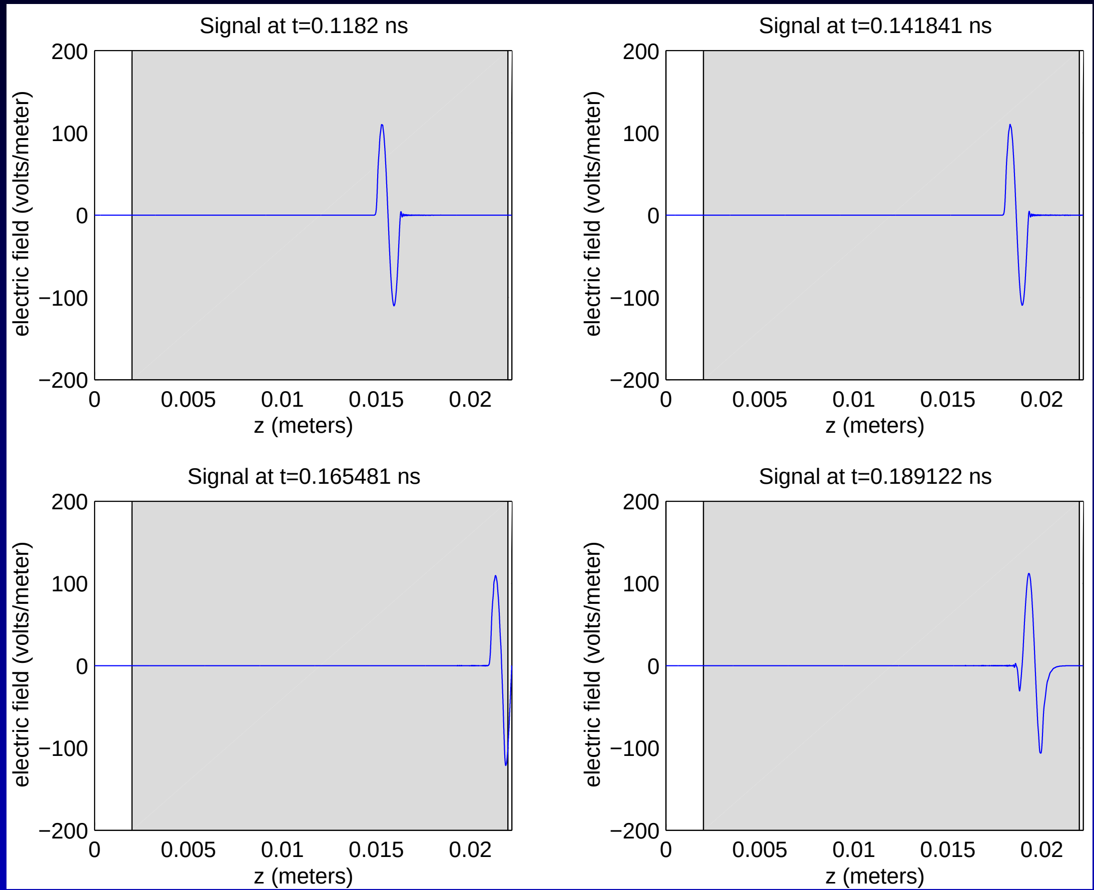
Numerical Discretization

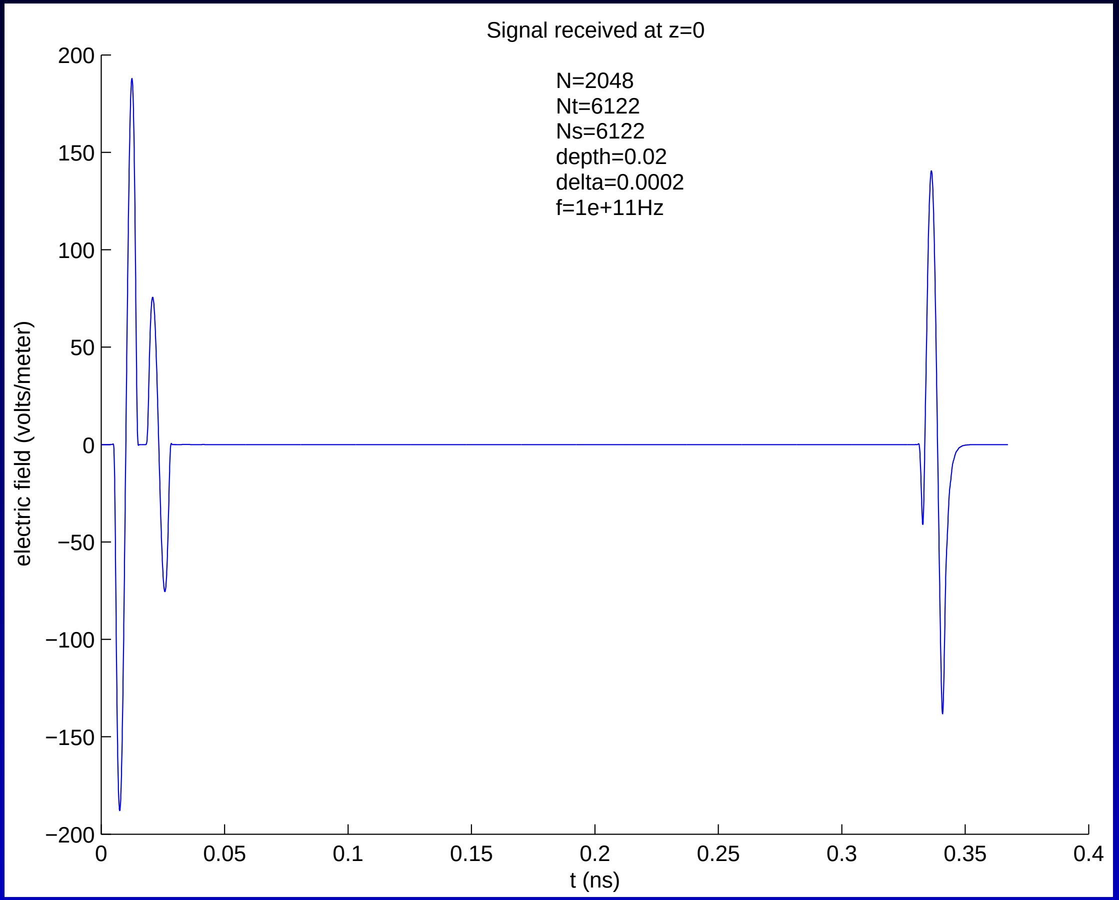
- Second order FEM in space
  - piecewise linear splines
- Second order FD in time
  - Crank-Nicholson (P)
  - Central differences (E)
  - $e_n \longrightarrow p_n \longrightarrow e_{n+1} \longrightarrow p_{n+1} \longrightarrow \cdots$
- E equation implicit, LU factorization used

Sample Problem



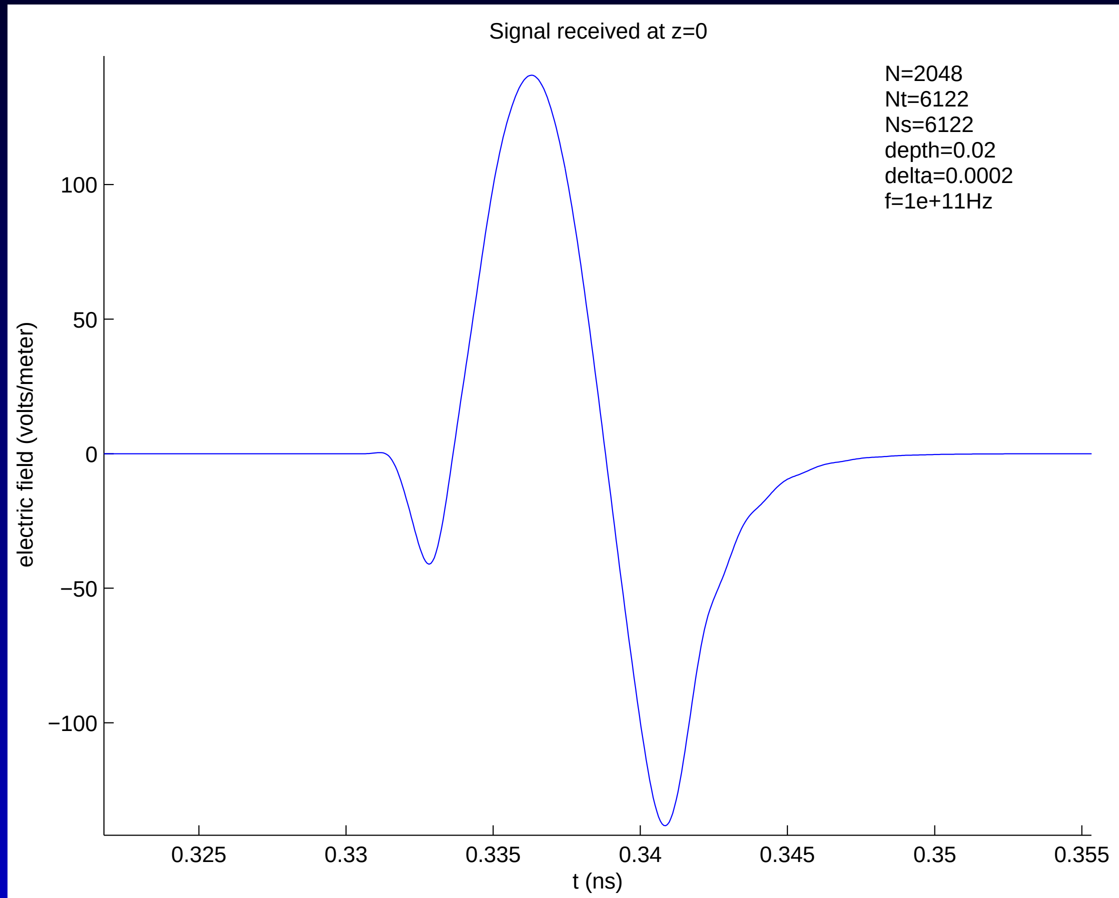
Computed solutions at different times of a windowed electromagnetic pulse at f=100GHz incident on a Debye medium with a crack δ=.0002m wide located d=.02m into the material.

Sample Problem (Cont.)



Reflected signal received at z=0.

Sample Problem (Cont.)



Close up look at reflected signal received at z=0
Shows "important" parts of the signal.

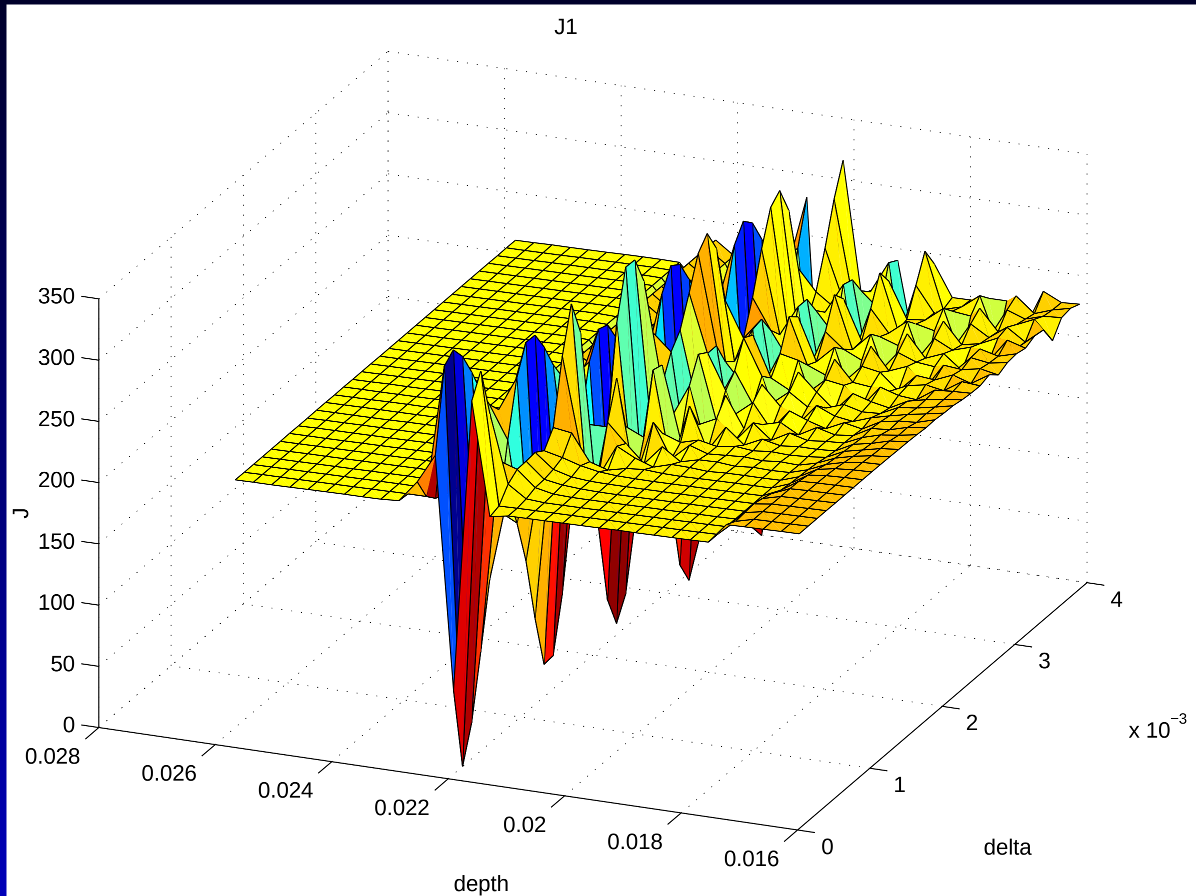
Gap Detection Inverse Problem

- Assume we have data, $\hat{E}_i$, recorded at z=0
- Given d and δ we can simulate the electric field
- Estimate d and δ by solving an inverse problem:

Find q=$(d, \delta) \in Q_{ad}$ such that the following objective function is minimized:

$$\mathcal{J}_1(q) = \frac{1}{2S} \sum_{i=1}^{S} |E(t_i, 0; q) - \hat{E}_i|^2.$$

$\mathcal{J}_1(q)$ Surface Plot



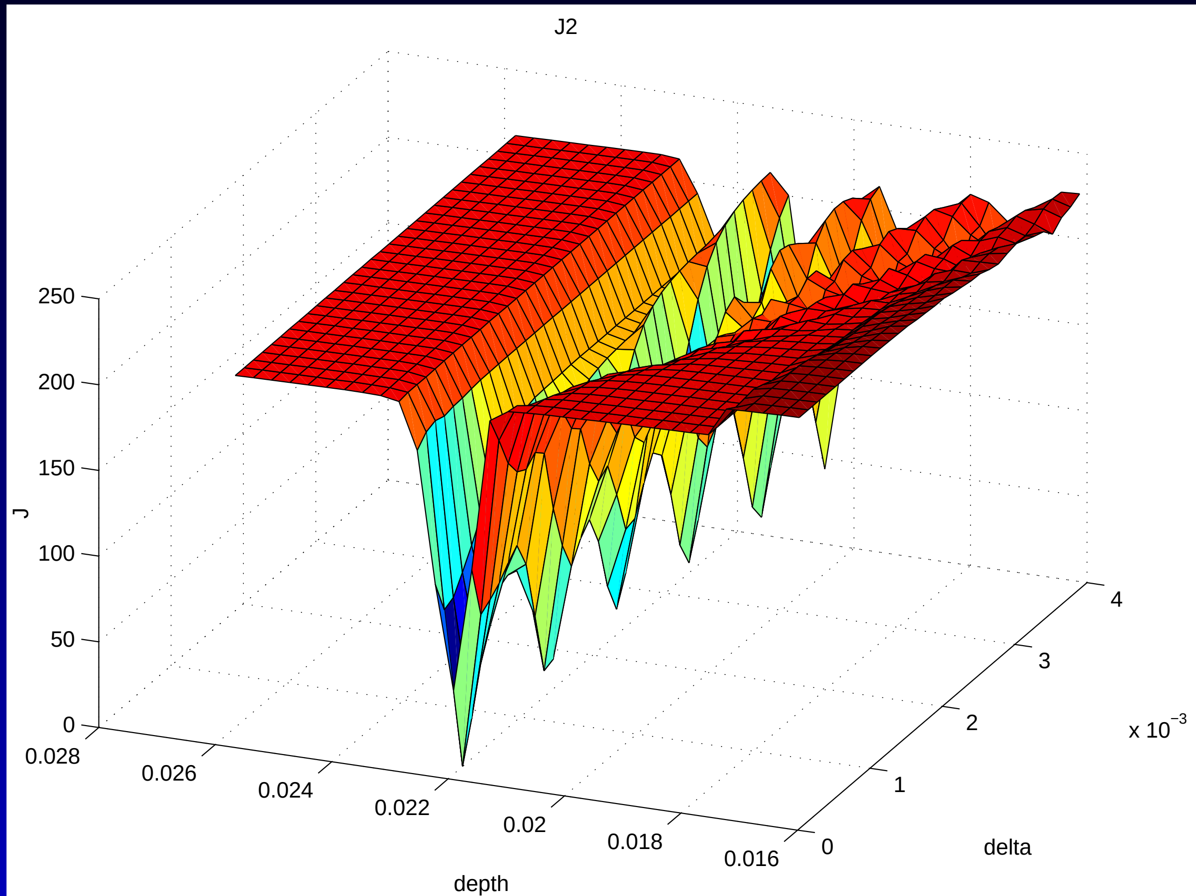
Surface plot of the Ordinary Least Squares objective function demonstrating peaks in $\mathcal{J}_1$, and exhibiting many local minima.

Improved Objective Function

Consider the following formulation of the Inverse Problem:
Find q=$(d, \delta) \in Q_{ad}$ such that the following objective function is minimized:

$$\mathcal{J}_2(q) = \frac{1}{2S} \sum_{i=1}^{S} \Big| |E(t_i, 0; q)| - |\hat{E}_i| \Big|^2 .$$

$\mathcal{J}_2(q)$ Surface Plot



Close up surface plot of our Modified Least Squares objective function demonstrating lack of peaks in $\mathcal{J}_2$, but still exhibiting many local minima.

Final Estimates (d)

d		δ = .0002	.0004	.0008
.02	(N=1024)	.0200022	.0200006	.0200002
.04	(N=2048)	.0399974	.0400005	.0399999
.08	(N=4096)	.0799987	.0800006	.0800003
.1	(N=8192)	.0999974	.1	.0999999
.2	(N=16384)	.200005	.2	.200001

Final Estimates (δ)

		δ		
d		.0002	.0004	.0008
.02	(N=1024)	.000196754	.000398642	.00079707
.04	(N=2048)	.000203916	.000394204	.000793622
.08	(N=4096)	.000202273	.000395791	.000794401
.1	(N=8192)	.000203876	.000396203	.000795985
.2	(N=16384)	.000191808	.00040297	.00080129

Comments on 1D Gap Problem

- Our modified Least Squares objective function "fixes" peaks in $\mathcal{J}$
- Can test on both sides of detected minima to ensure global minimization
- We are able to detect a $.2mm$ wide crack behind a $20cm$ deep slab
- Even adding random noise (equivalent to 20% relative noise) does not significantly hinder our inverse problem solution method

2D Problem Outline

- Model
  - Equations
  - Boundary Conditions
- Computational Methods
- Sample Forward Simulations
- Inverse Problem

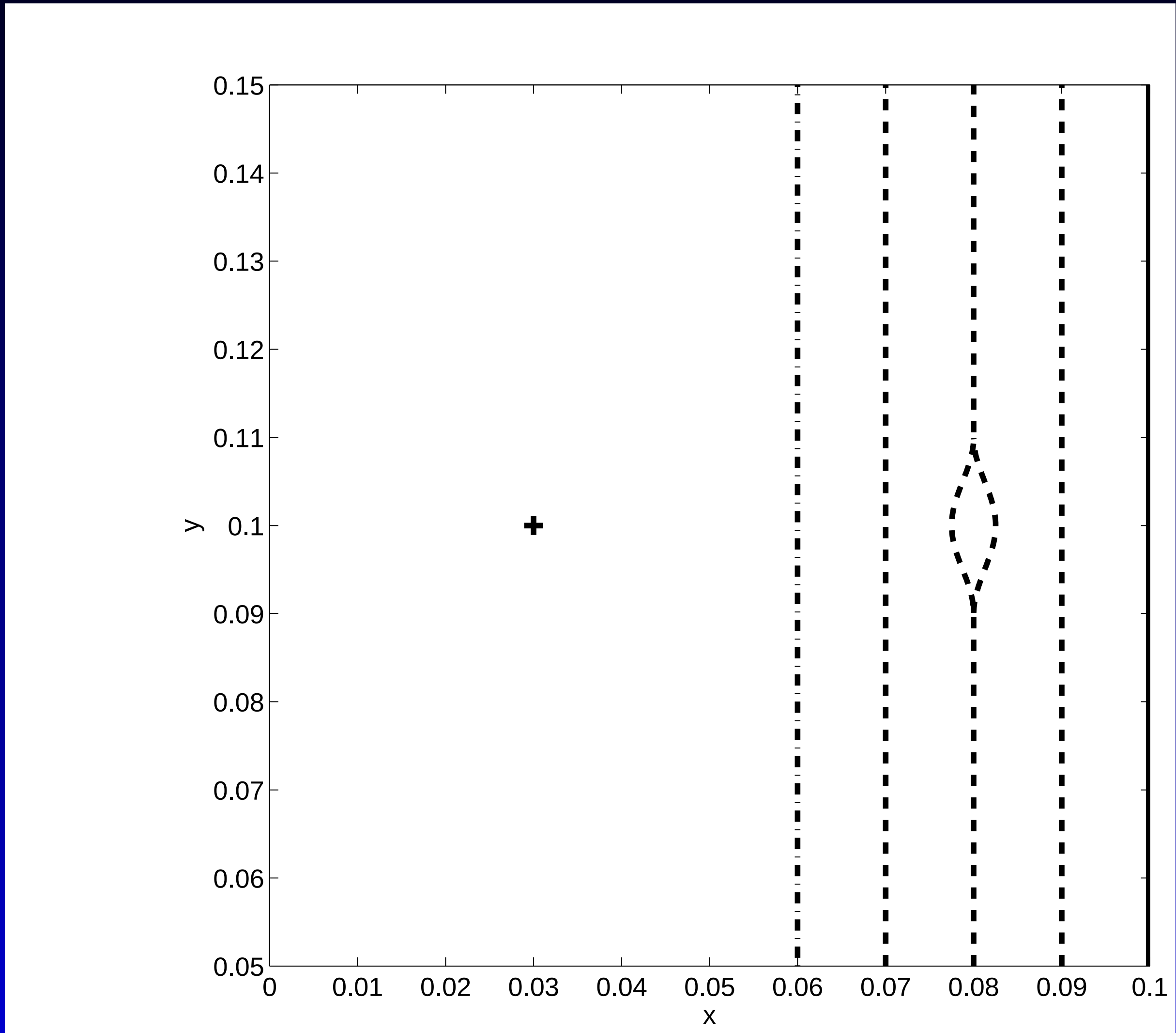
Voids in Foam

The foam on the space shuttle is sprayed on in layers (thus the acronym SOFI). Voids occur between layers.

Cured Layer

As the top of each layer cures, a thin knit line is formed which is of higher density (i.e., is comprised of smaller, more tightly packed polyurethane cells).

Sample Domain with Void



Dashed lines represent knit lines, dot-dash is foam/air interface. Elliptical pocket (5 mm) between knit lines is a void. “+” marks the signal receiver. Back wall is perfect conductor.

Simplifications

- Assume single-cycle pulse of fixed frequency
  - Possibly only tracking peak frequency
  - Possibly solving broadband problem in parallel
  - Maxwell's equations reduce to wave equation
- Assume homogenized material
  - For low frequency, microstructure is negligible
  - For fixed frequency, single wave speed
  - Possibly from homogenization method
- Assume 2D (uniformity in third)

2D Wave Equation

We assume the electric field to be polarized in the z direction, thus for $\vec{E} = (0, 0, E)$ and $\vec{x} = (x, y)$

$$\epsilon(\vec{x}) \frac{\partial^2 E}{\partial t^2}(t, \vec{x}) - \nabla \cdot \left(\frac{1}{\mu(\vec{x})} \nabla E(t, \vec{x}) \right) = -\frac{\partial J_s}{\partial t}(t, \vec{x})$$

where $\epsilon(\vec{x})$ and $\mu(\vec{x}) = \mu_0$ are the dielectric permittivity and permeability, respectively.

$$J_s(t, \vec{x}) = \delta(x) e^{-((t-t_0)/t_0)^4},$$

where $t_0 = t_f/4$ when t_f is the period of the interrogating pulse.

Boundary Conditions

Consider $\Omega = [0, 0.1] \times [0, 0.2]$

- Reflecting (Dirichlet) boundary conditions (right)

$$[E]_{x=0.1} = 0$$

- First order absorbing boundary conditions (left)

$$\frac{\partial E}{\partial t} - \sqrt{\frac{1}{\epsilon(\vec{x})\mu_0}} \frac{\partial E}{\partial x}\bigg|_{x=0} = 0$$

- Symmetric boundary conditions (top and bottom)

$$\left[\frac{\partial E}{\partial y}\right]_{y=0, y=0.2} = 0$$

We use homogeneous initial conditions $E(0, \vec{x}) = 0$.

Modeling Knit Lines/Void

- The speed of propagation in the domain is

$$c(\vec{x}) = \frac{c_0}{n(\vec{x})} = \sqrt{\frac{1}{\epsilon(\vec{x})\mu_0}},$$

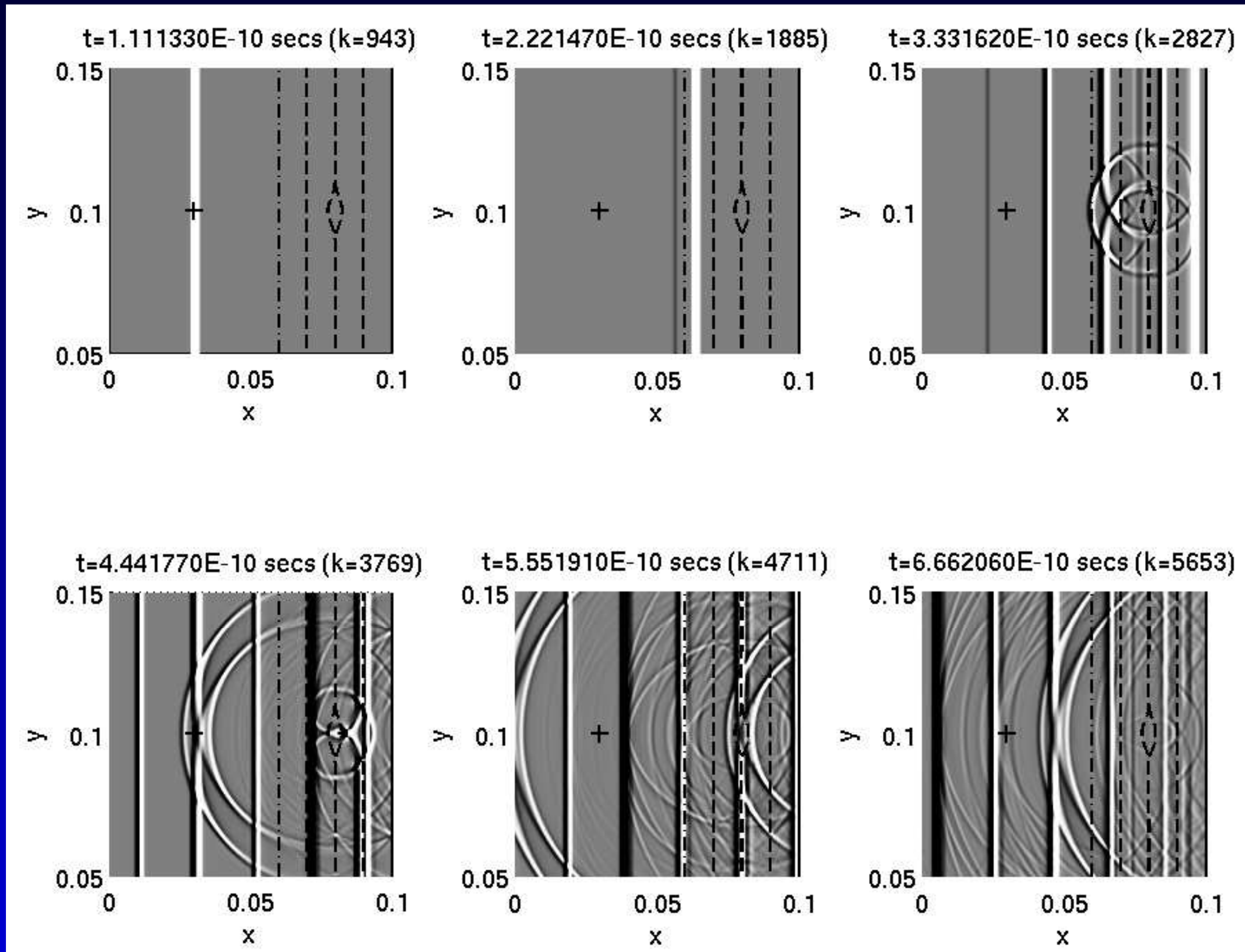
  where c_0 is the speed in a vacuum and n is the index of refraction.

- We may model knit lines or a void by changing the index of refraction, thus effectively the speed in that region.

2D Numerical Discretization

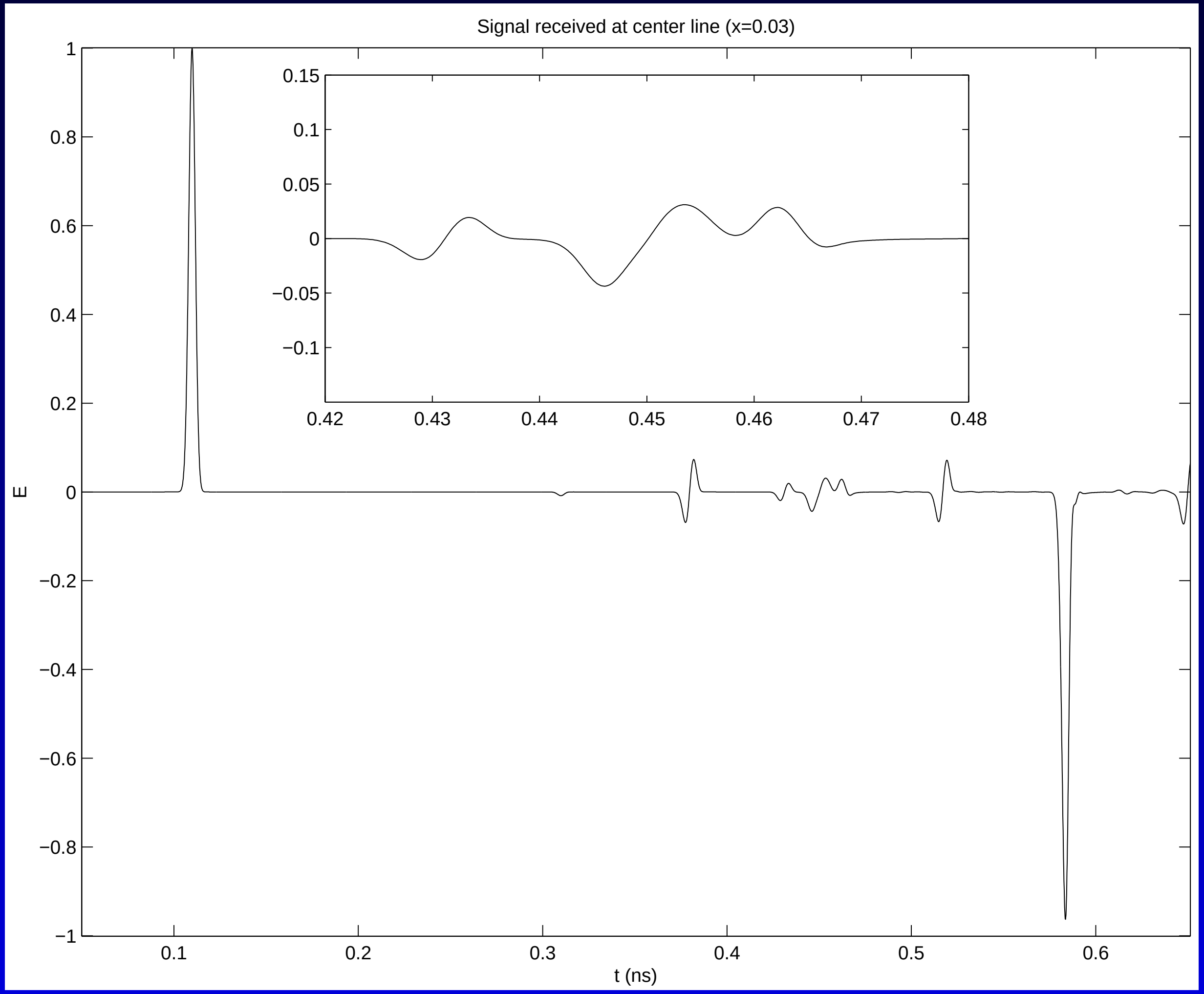
- Second order (piecewise linear) FEM in space
- Second order (centered) FD in time
- Linear solve (sparse)
  - Preconditioned conjugate-gradient (matrix-free)
  - LU factorization
  - **Mass lumping (explicit)**
- Stair-stepping for non-rectilinear interfaces

Plane Wave Simulation



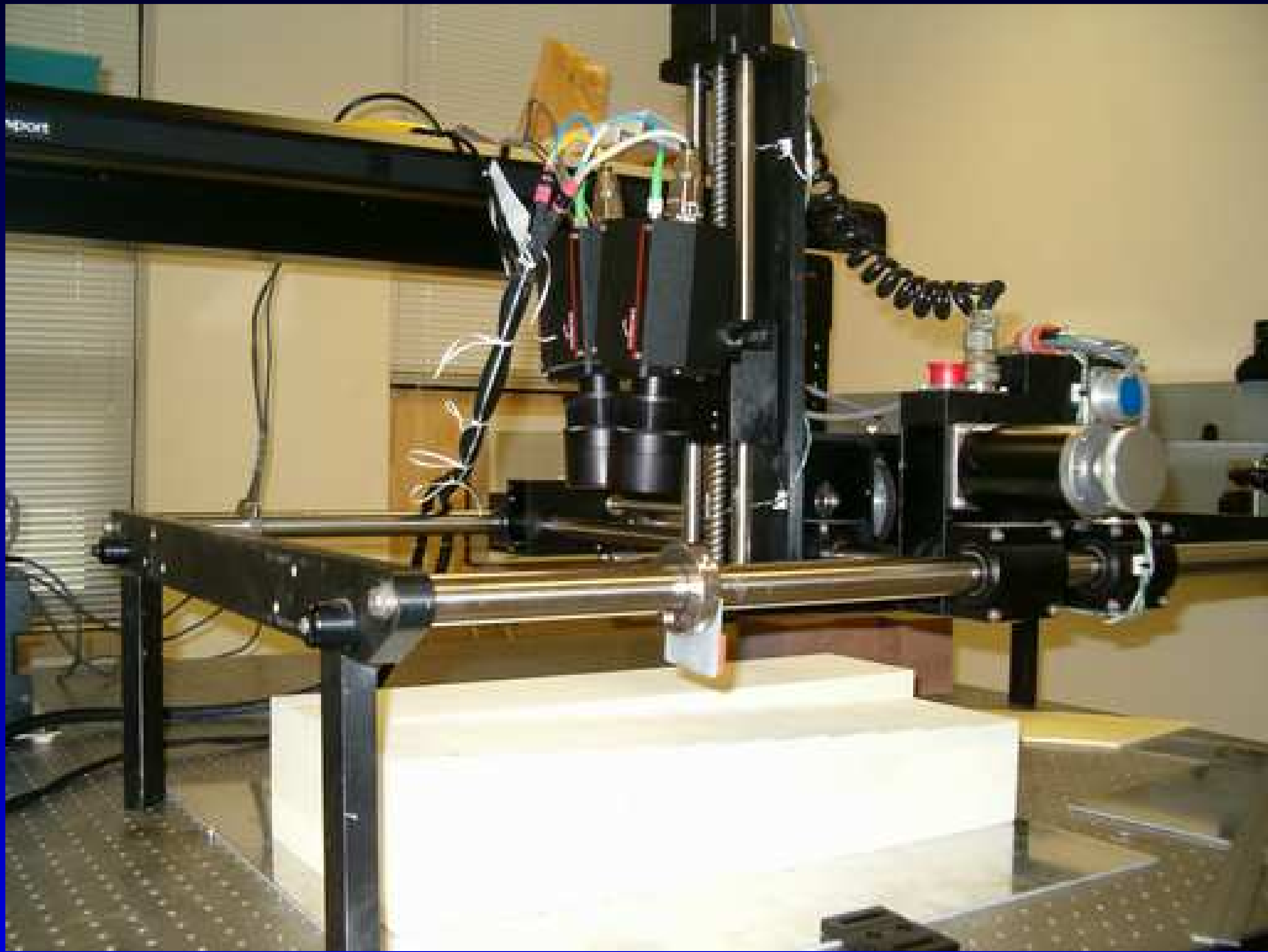
Source located at $x = 0$, receiver at $x = 0.03$.

Plane Wave Signal



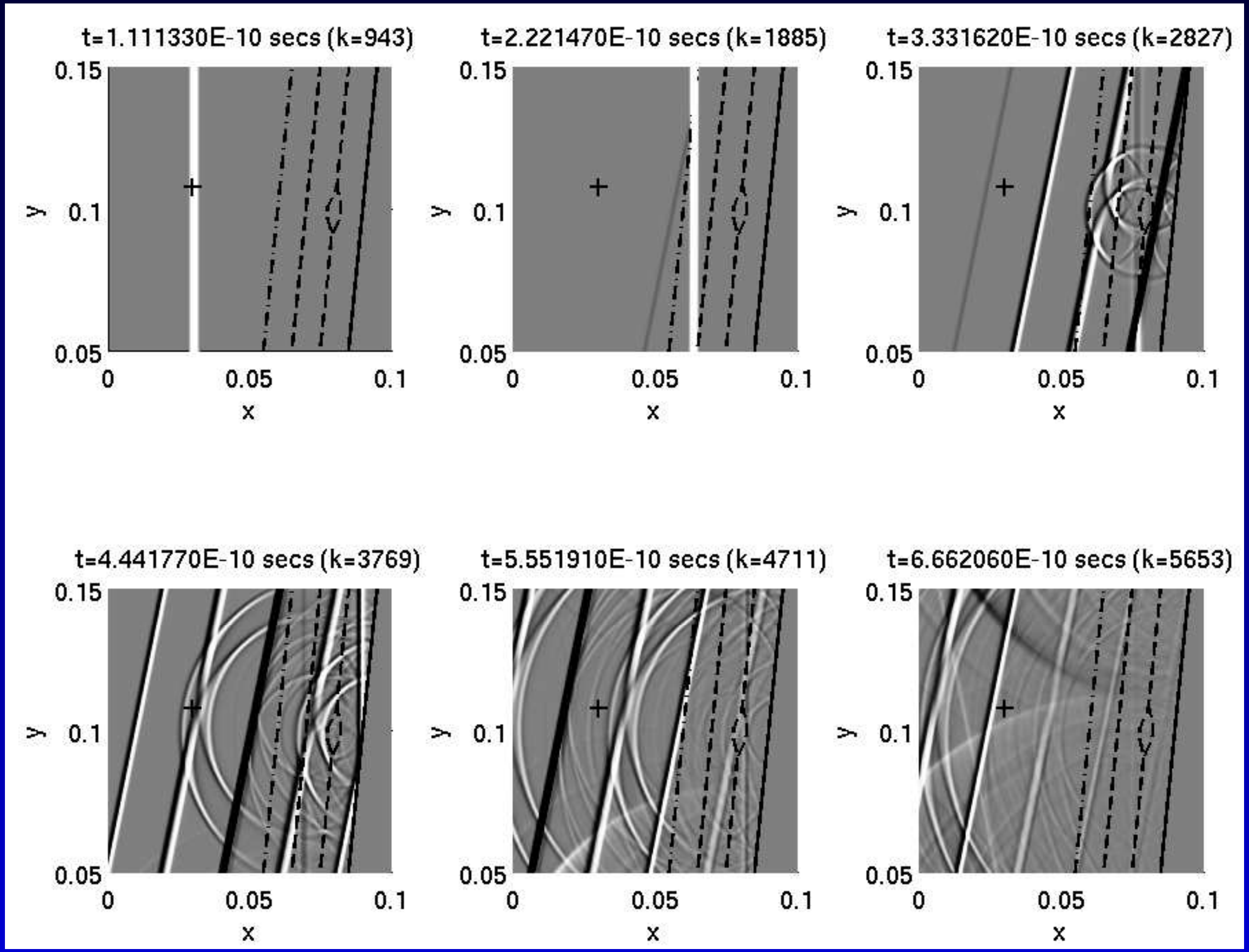
Interrogating signal simulates a sine curve truncated after one half period.
Reflections off void are shown in inset.

Picometrix T-Ray Setup



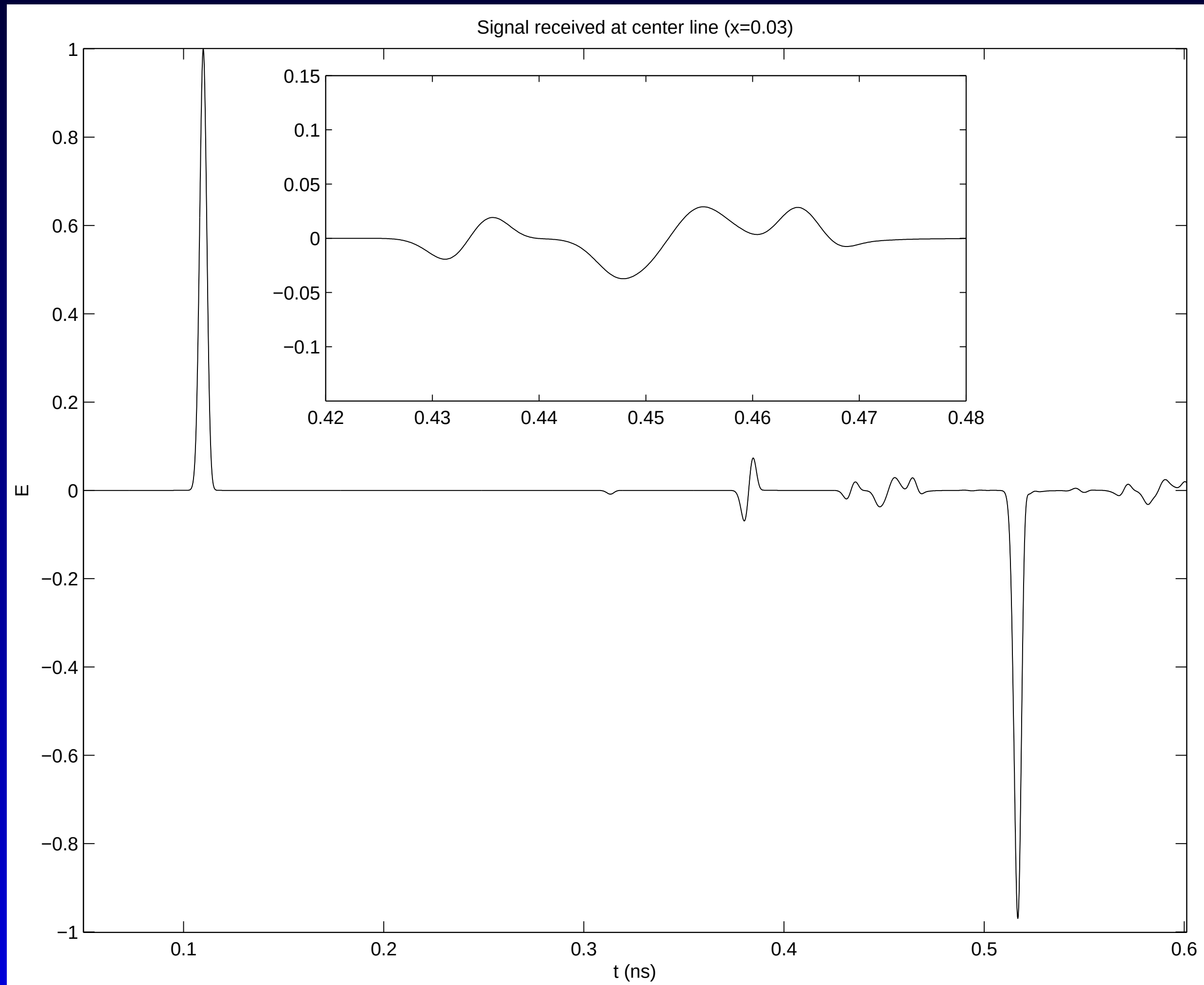
Note the non-normal incidence and ability to focus.

Oblique Plane Wave Simulation



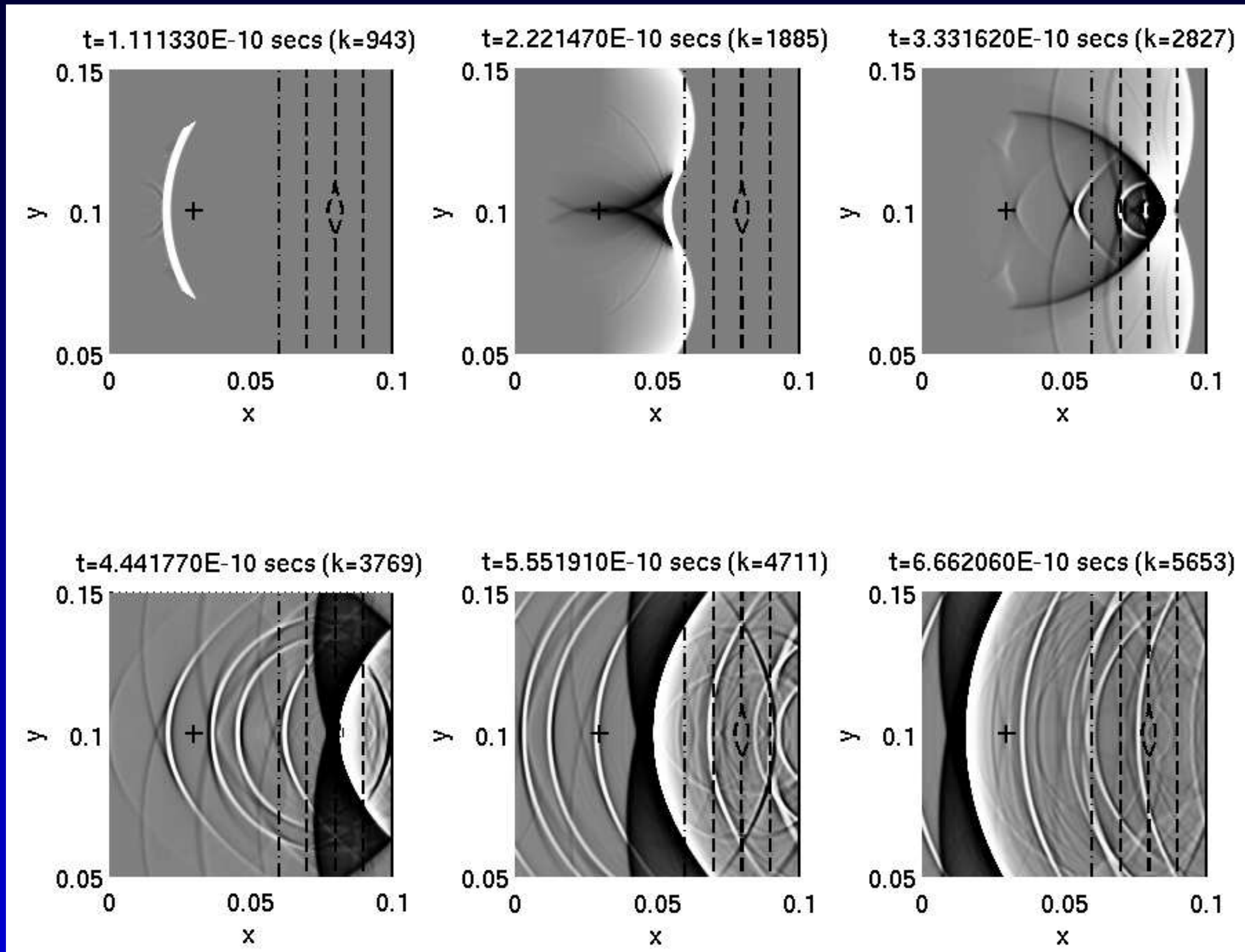
Source located at $x = 0$, receiver at $x = 0.03$, but raised to collect center of plane wave reflection.

Oblique Plane Wave Signal



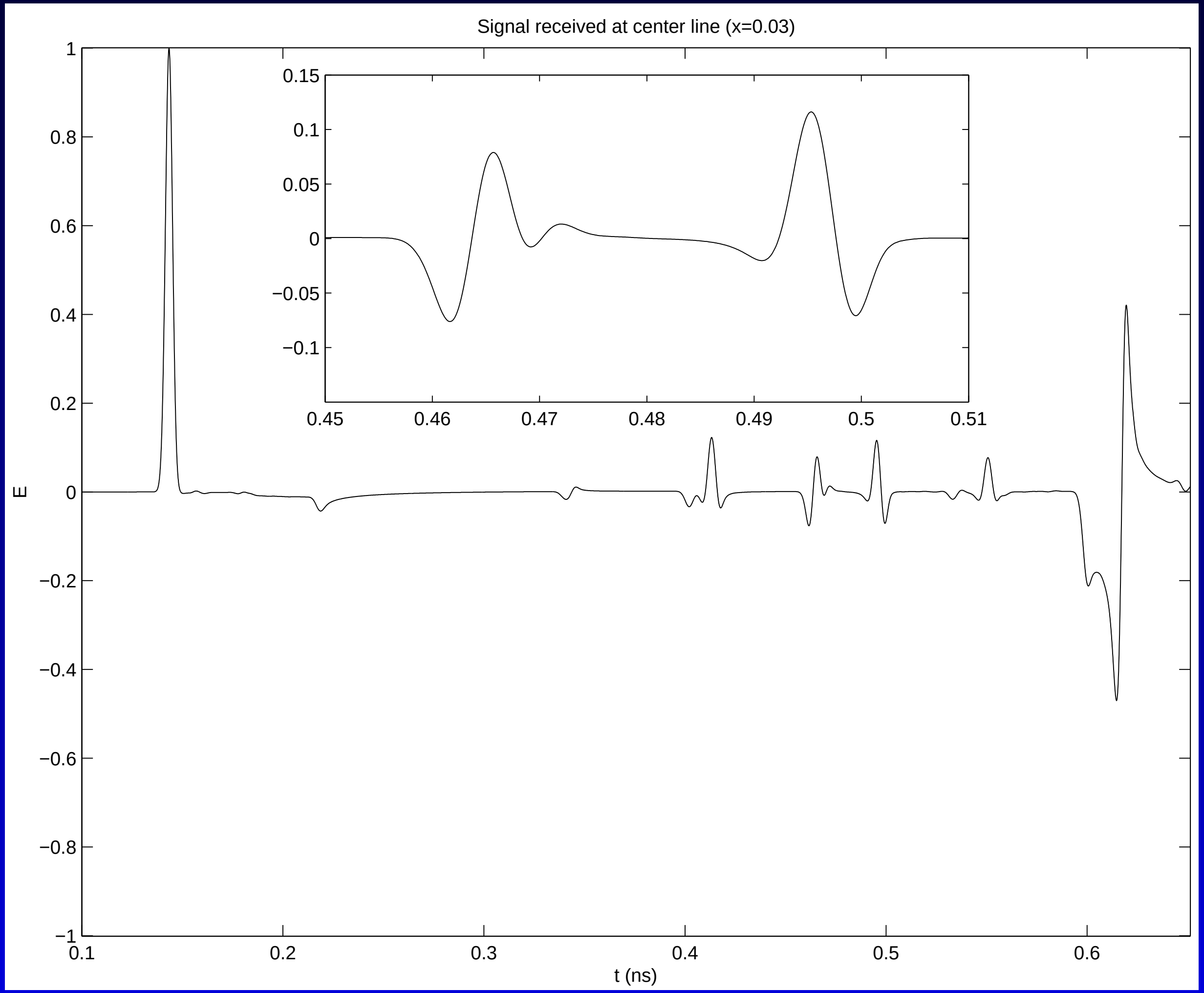
Nearly all of original signal returns even with an oblique angle of incidence. (Note: last knit line removed.)

Focused Wave Simulation



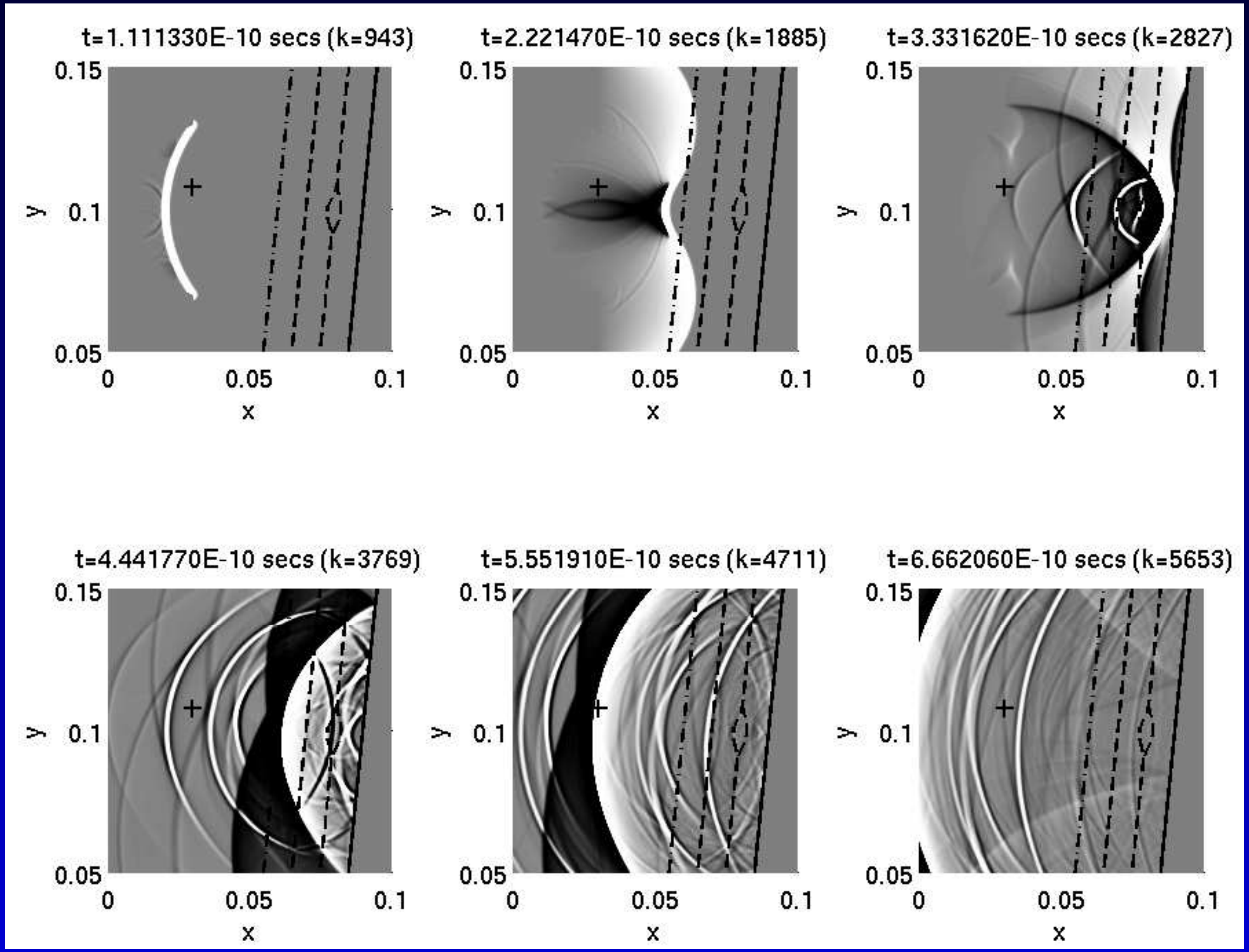
Source modeled using scattered field formulation of point source reflected from elliptical mirror. Receiver located at $x = 0.03$. Note top and bottom boundary conditions are now absorbing.

Focused Wave Signal



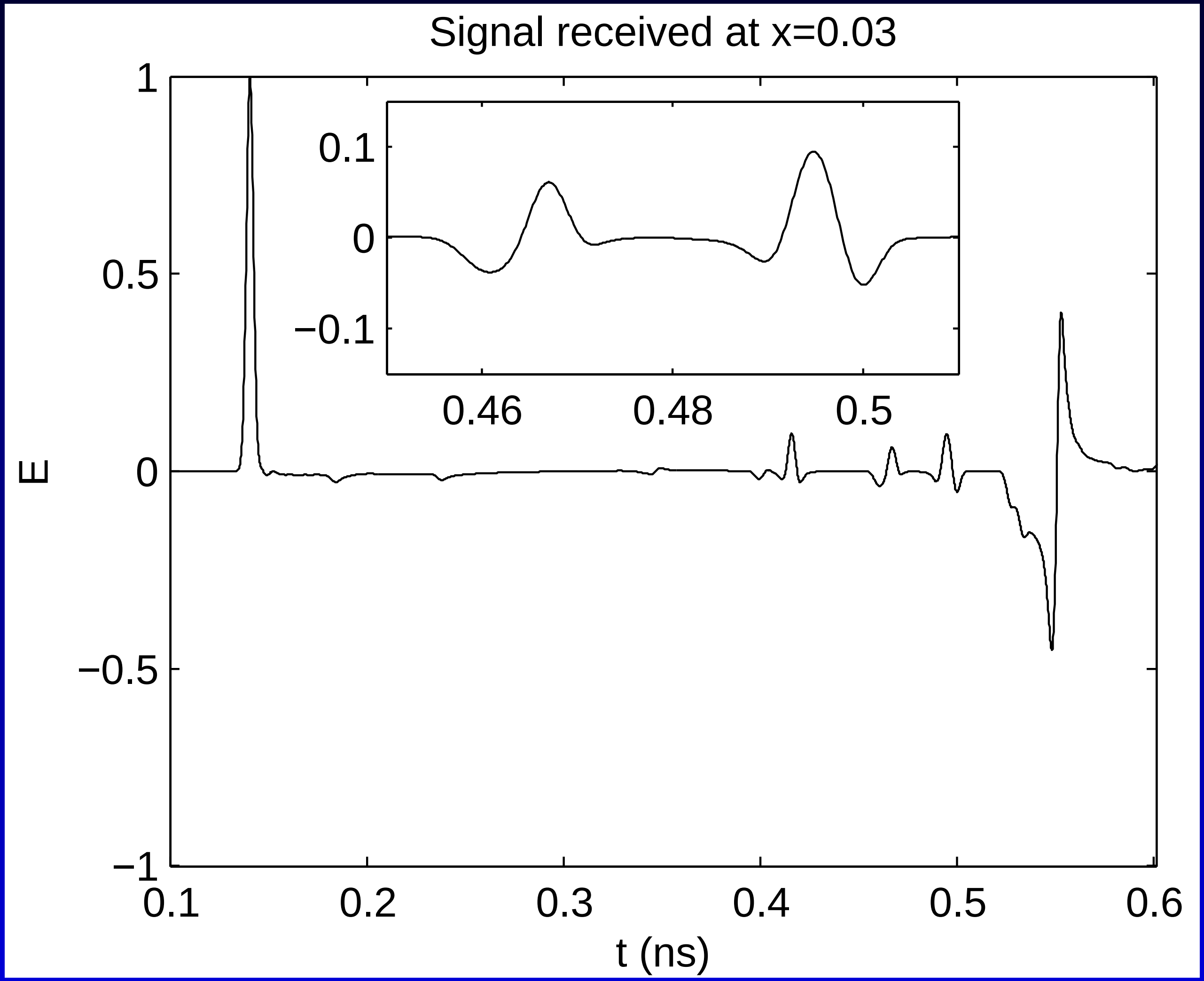
Although reflections off of void are larger, the total energy that returns is less than the plane wave simulation.

Oblique Focused Wave



Source modeled using scattered field formulation of point source reflected from elliptical mirror. Receiver located at $x = 0.03$, but raised to collect center of focused wave reflection.

Oblique Focused Wave Signal



Data received from non-normally incident, focused wave. Reflection from void is similar in magnitude to normal incidence focused wave.

2D Void Inverse Problem

- Assume we have data, $\hat{E}_i$ at times t_i and $\mathbf{x} = \mathbf{x}^+$
- Given the width of an elliptical void w, we can simulate the electric field
- Estimate void width w by solving an inverse problem:

Find $w \in Q_{ad}$ such that the following objective function is minimized:

$$\mathcal{J}_1(w) = \frac{1}{2S} \sum_{i=1}^{S} |E(t_i, \mathbf{x}^+; w) - \hat{E}_i|^2.$$

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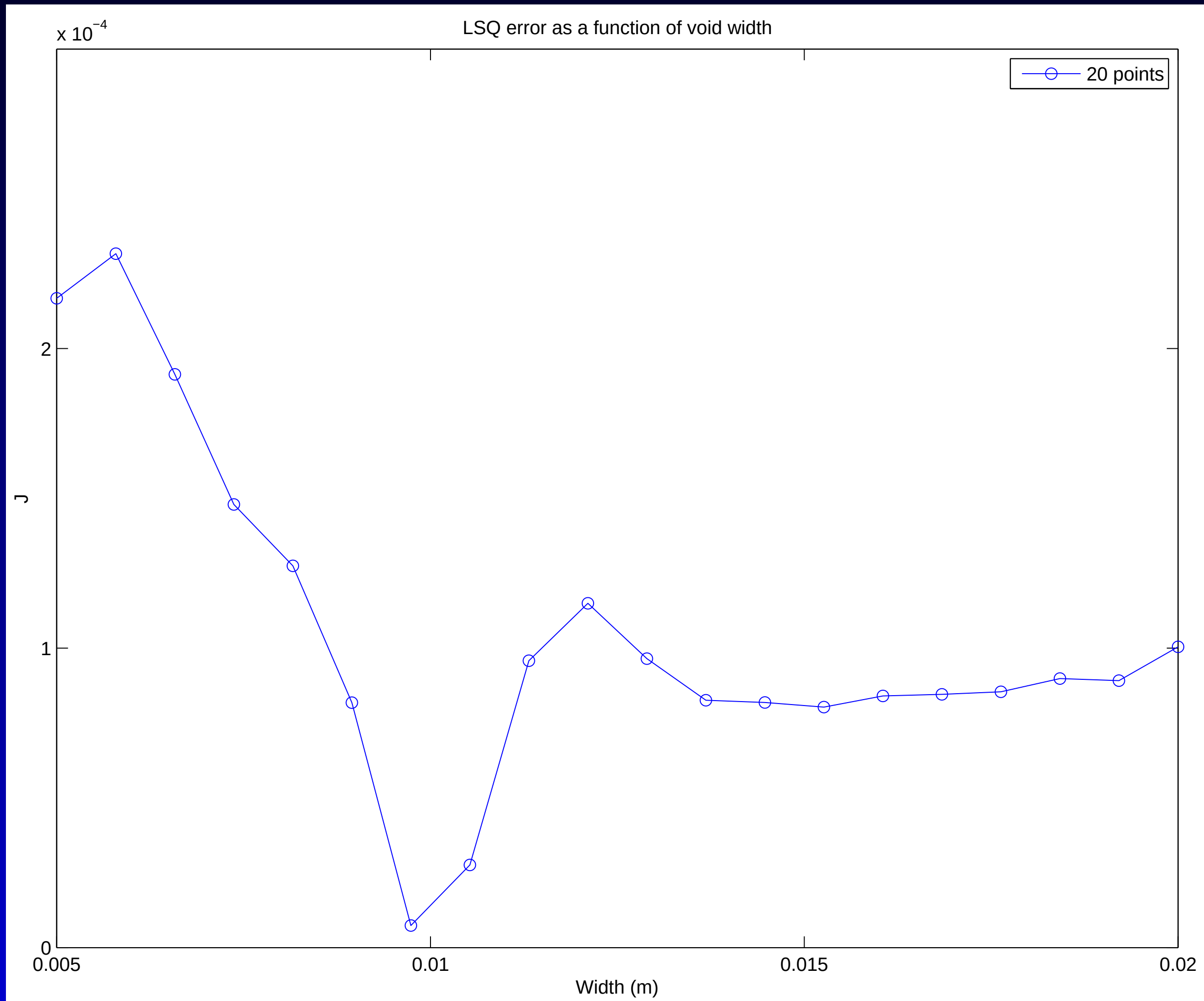
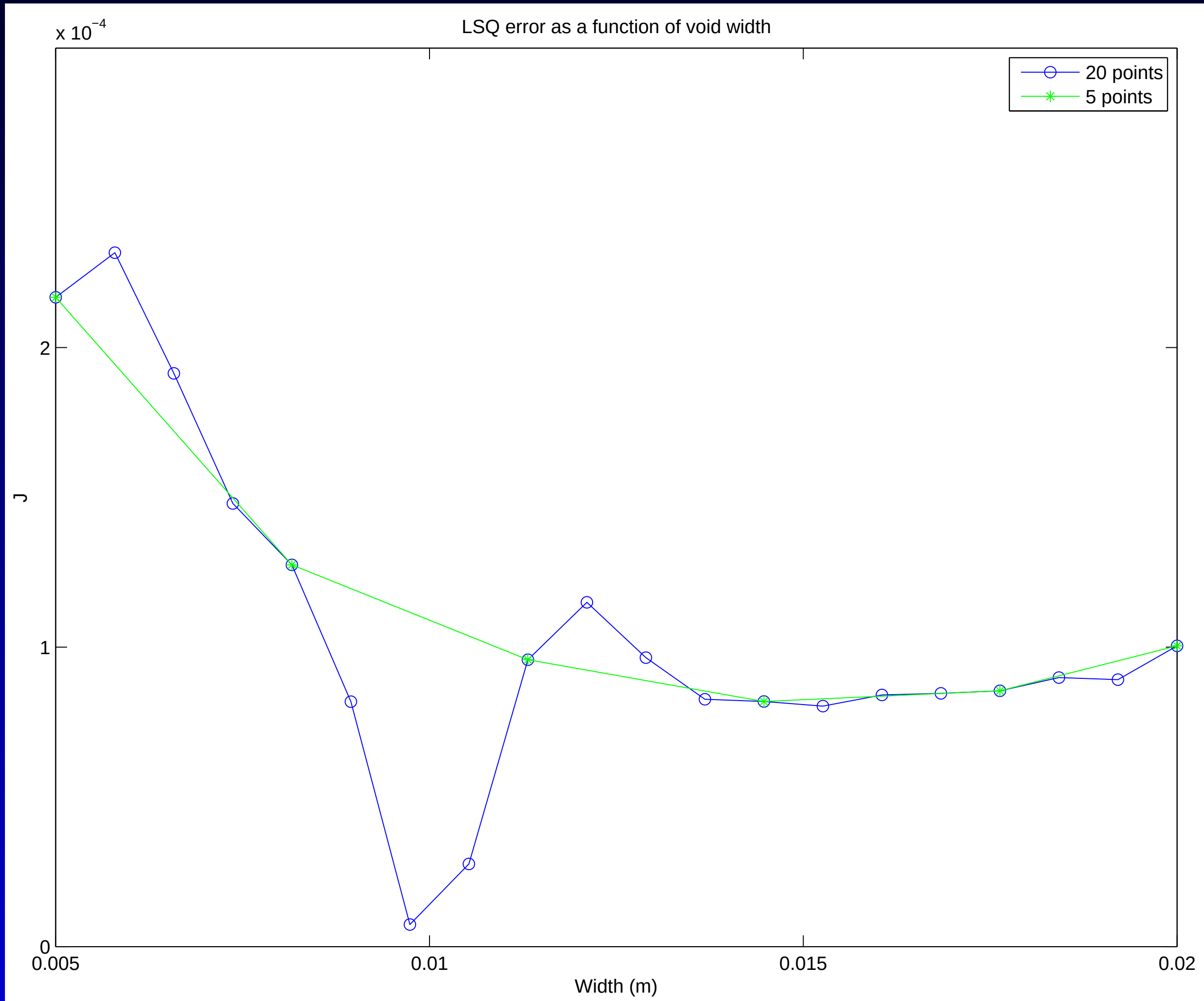
$\mathcal{J}_1$ – Objective Function



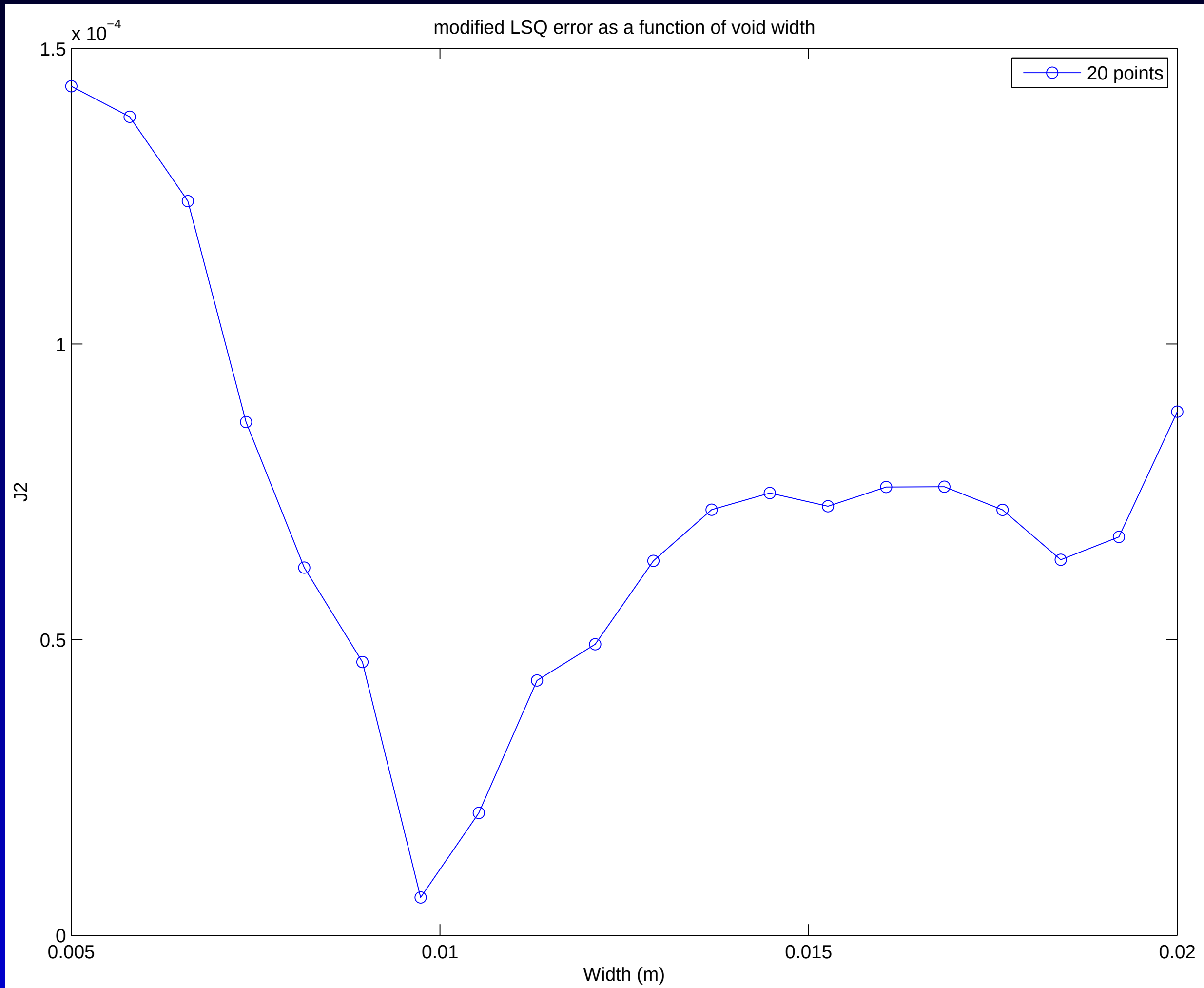
Location of the initial guess is crucial to minimizing with a gradient-based method.

$\mathcal{J}_1$ – Objective Function



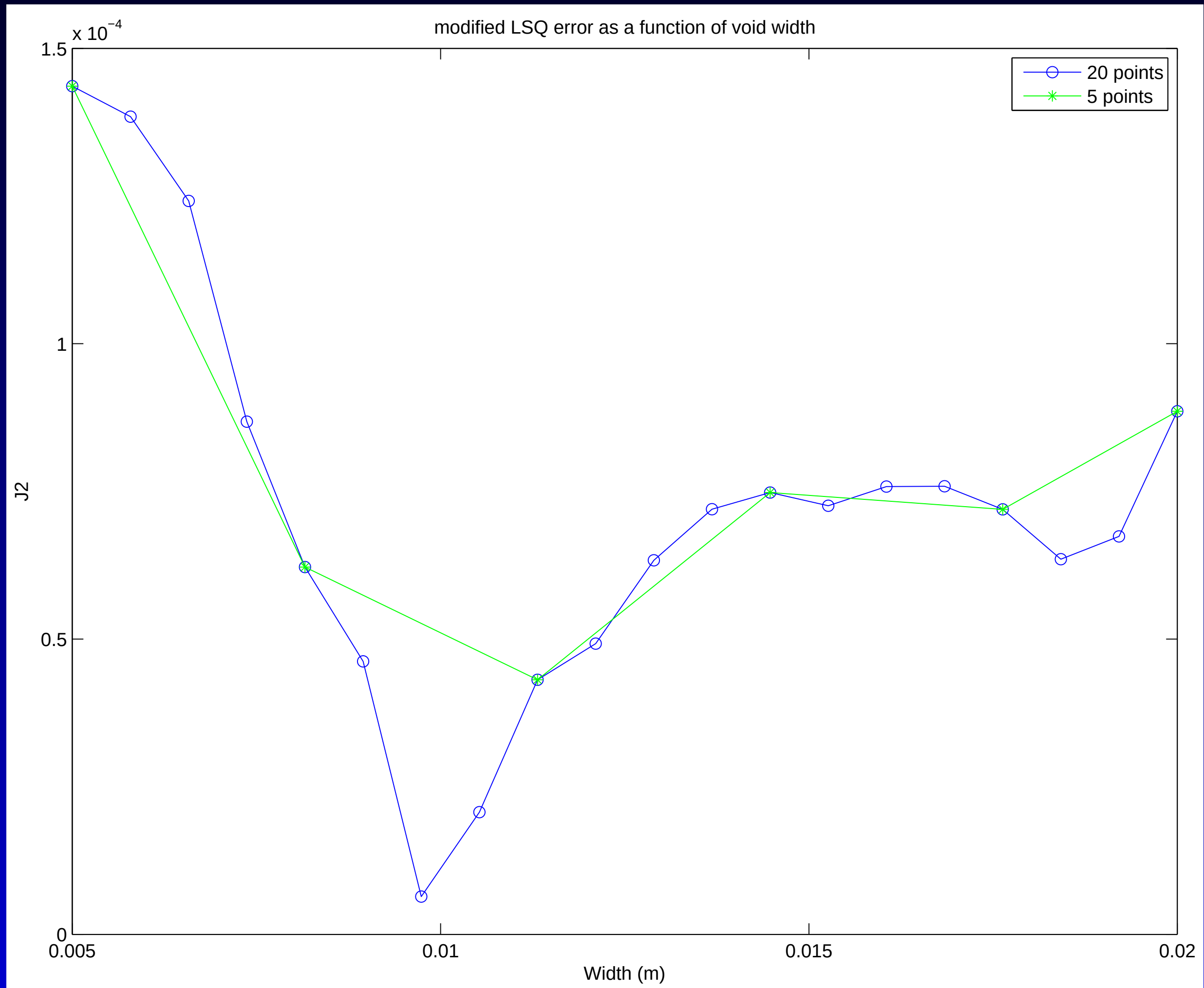
If we had only sampled the landscape with five points, we would have chosen poorly.

Improved Objective Function



$$\mathcal{J}_2(w) = \frac{1}{2S} \sum_{i=1}^{S} \left| |E(t_i, \mathbf{x}^+; w)| - |\hat{E}_i| \right|^2 .$$

Improved Objective Function

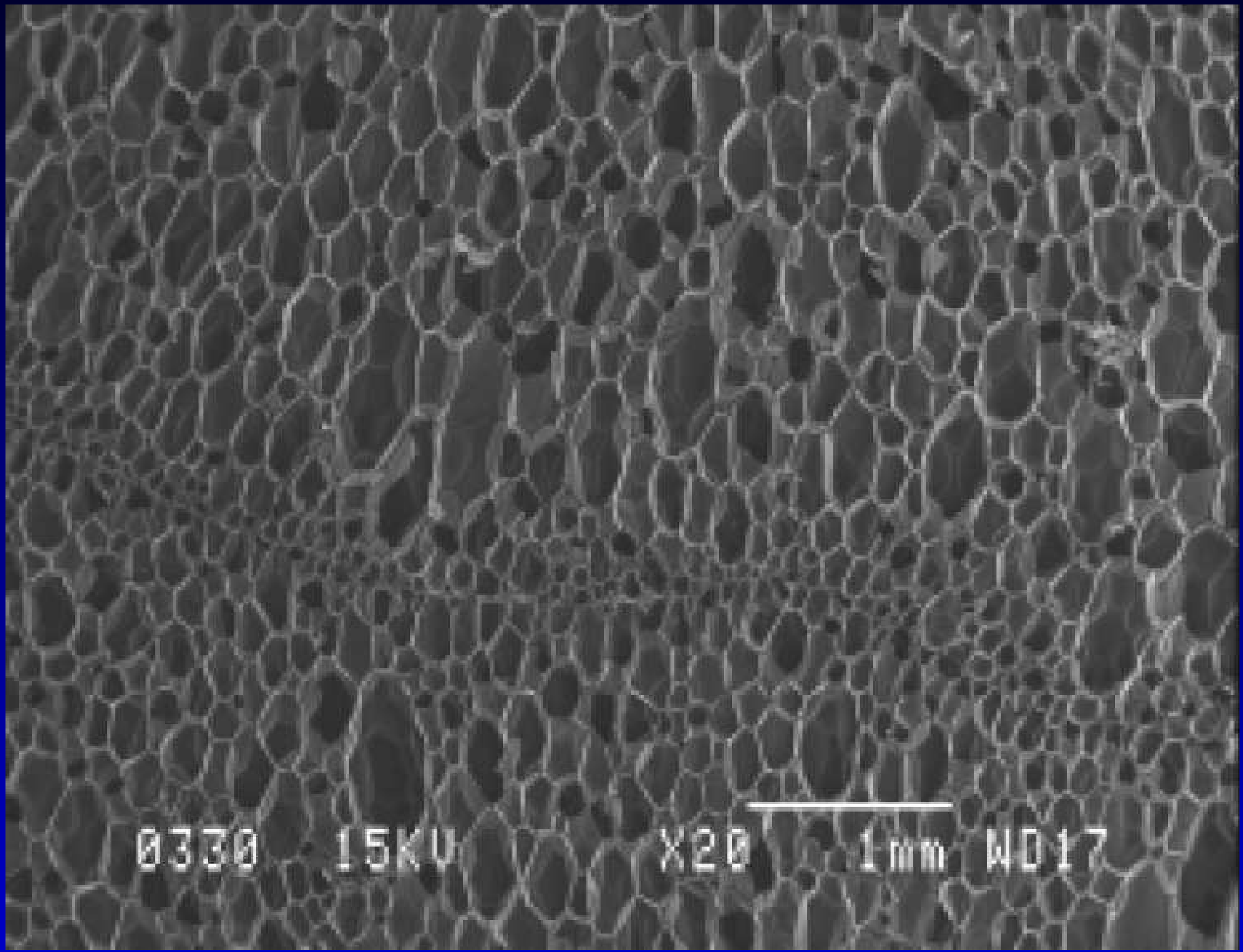


Better representation of overall agreement of signals; more forgiving in choosing initial guess.

2D Inverse Problem Results

- LM converges to minimum of $\mathcal{J}_2$ after 12 iterations
- Each forward solve is 1.5 hours
- This does not incorporate noise (SNR $\approx$ 100:1)

SOFI Under 20X Magnification



- Wavelength is on the order of 1mm; microstructure is smaller.
- Most loss in the material is due to scattering from faces.
- Modeling geometry of microstructure will help understand bulk effects.

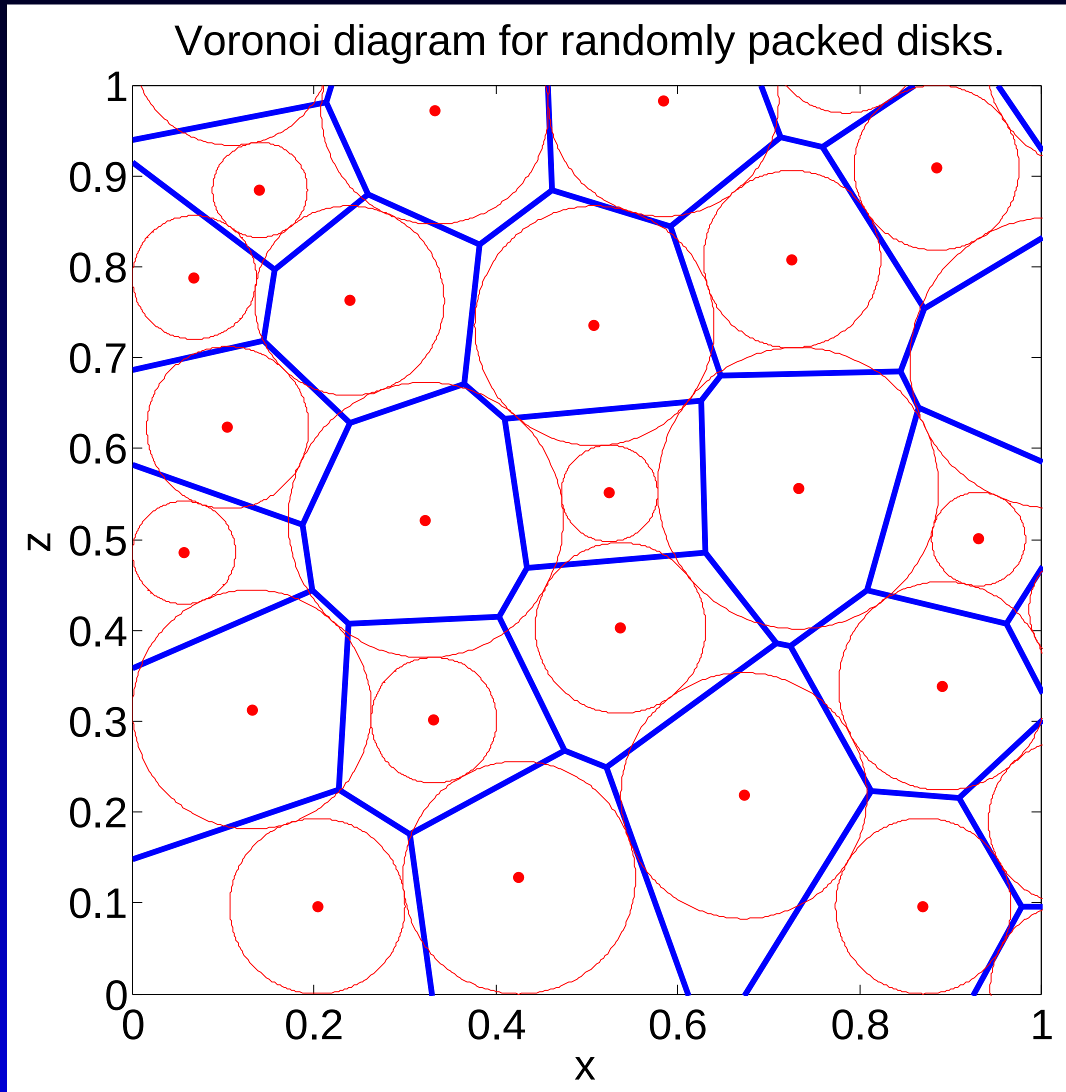
Microstructure

Model random microstructures:

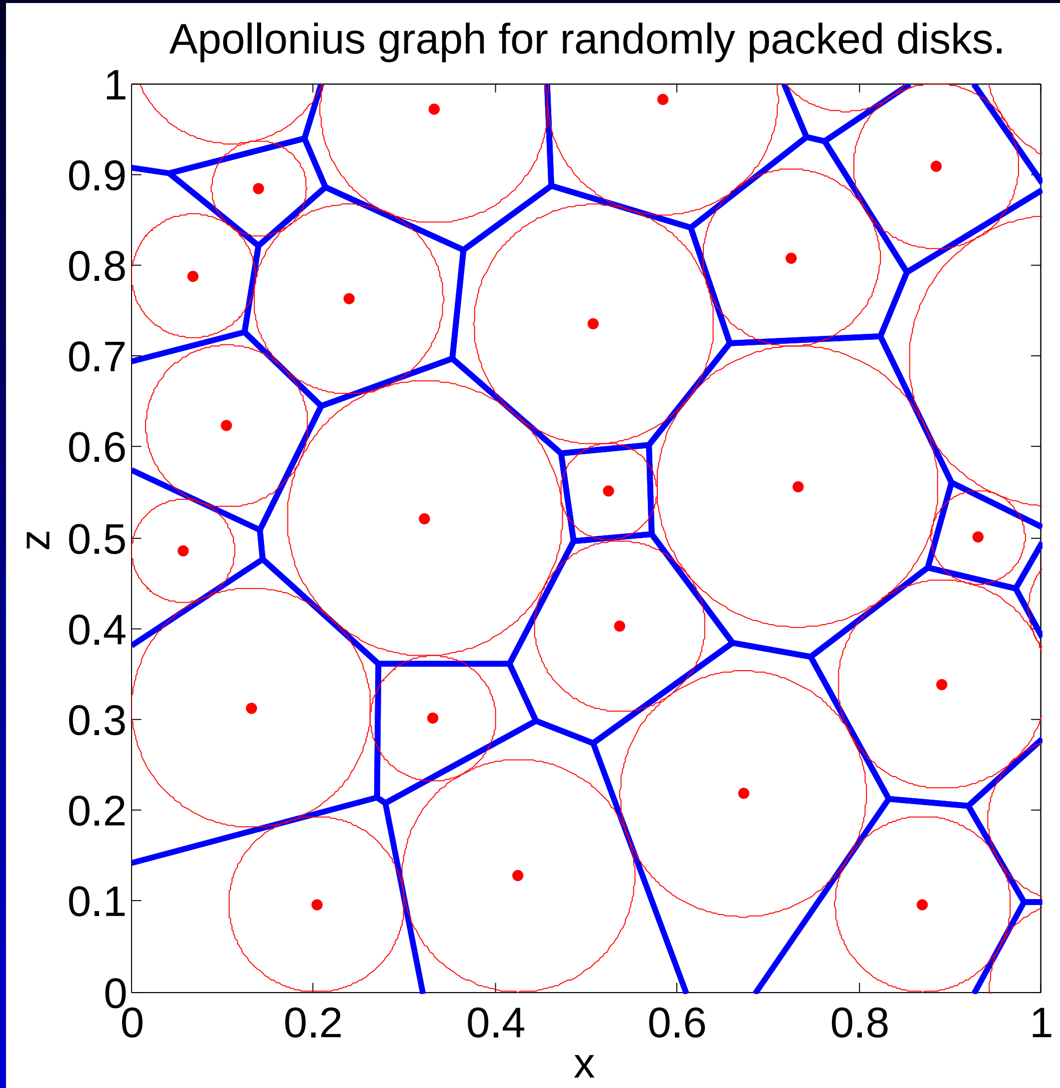
- “Random raindrops” algorithm
- Apollonius tessellation
- Constant wave speed

Note: Distributions of statistical parameters yields heterogeneity

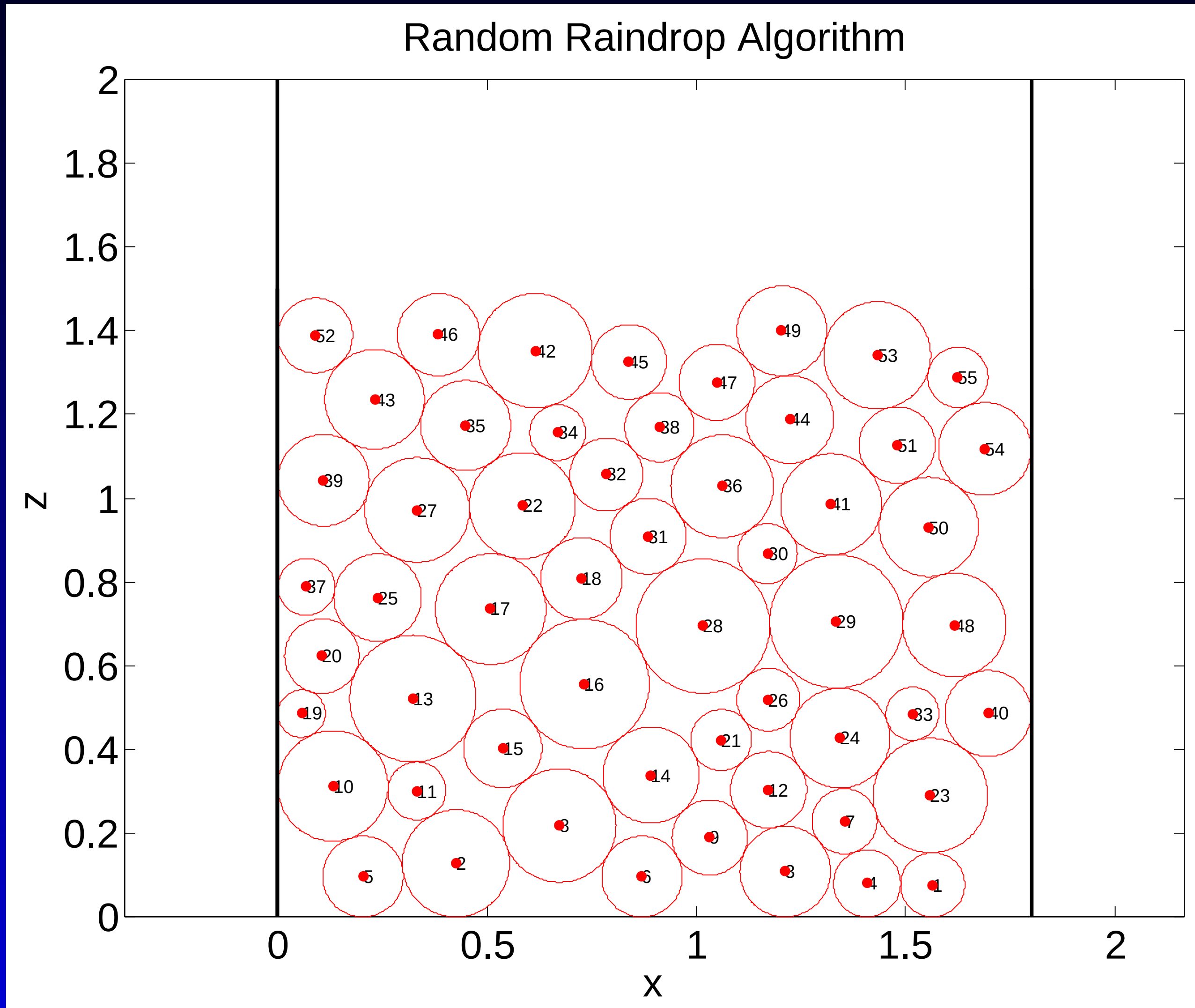
Voronoi Graph



Apollonius Graph

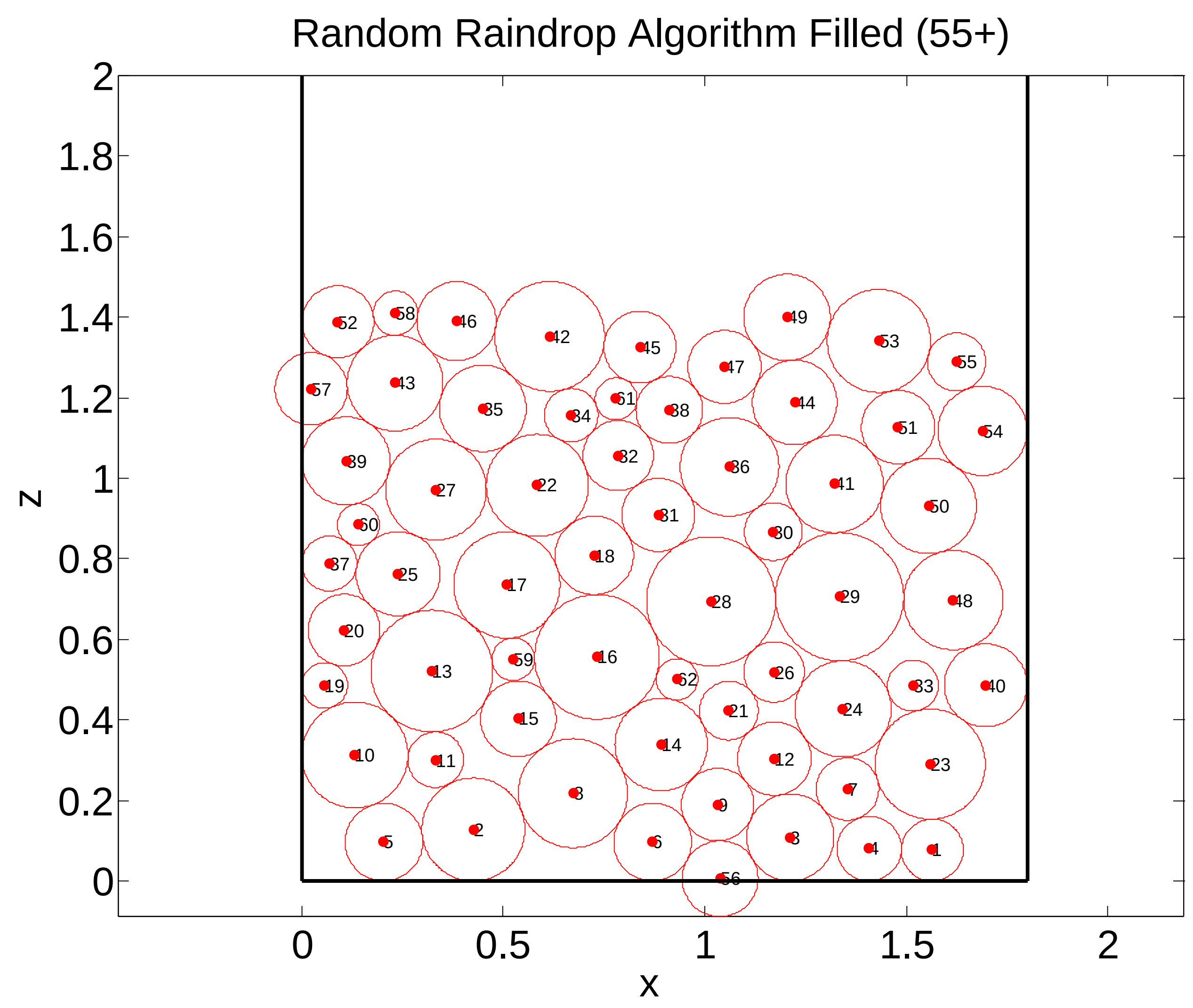


Random Raindrop



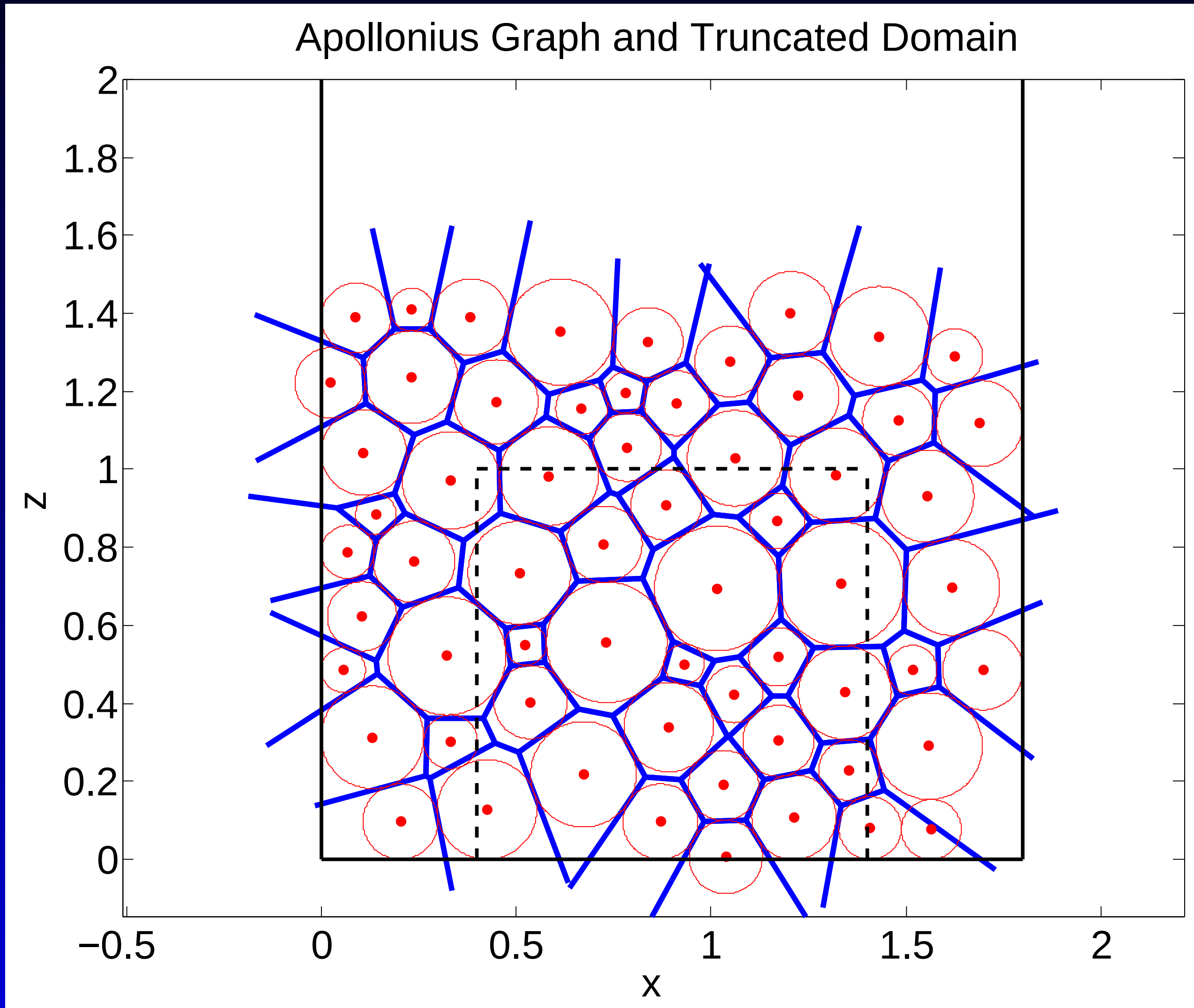
An example of the random raindrop or "drop and roll" algorithm for generating randomly close packed disks.

Filled-in



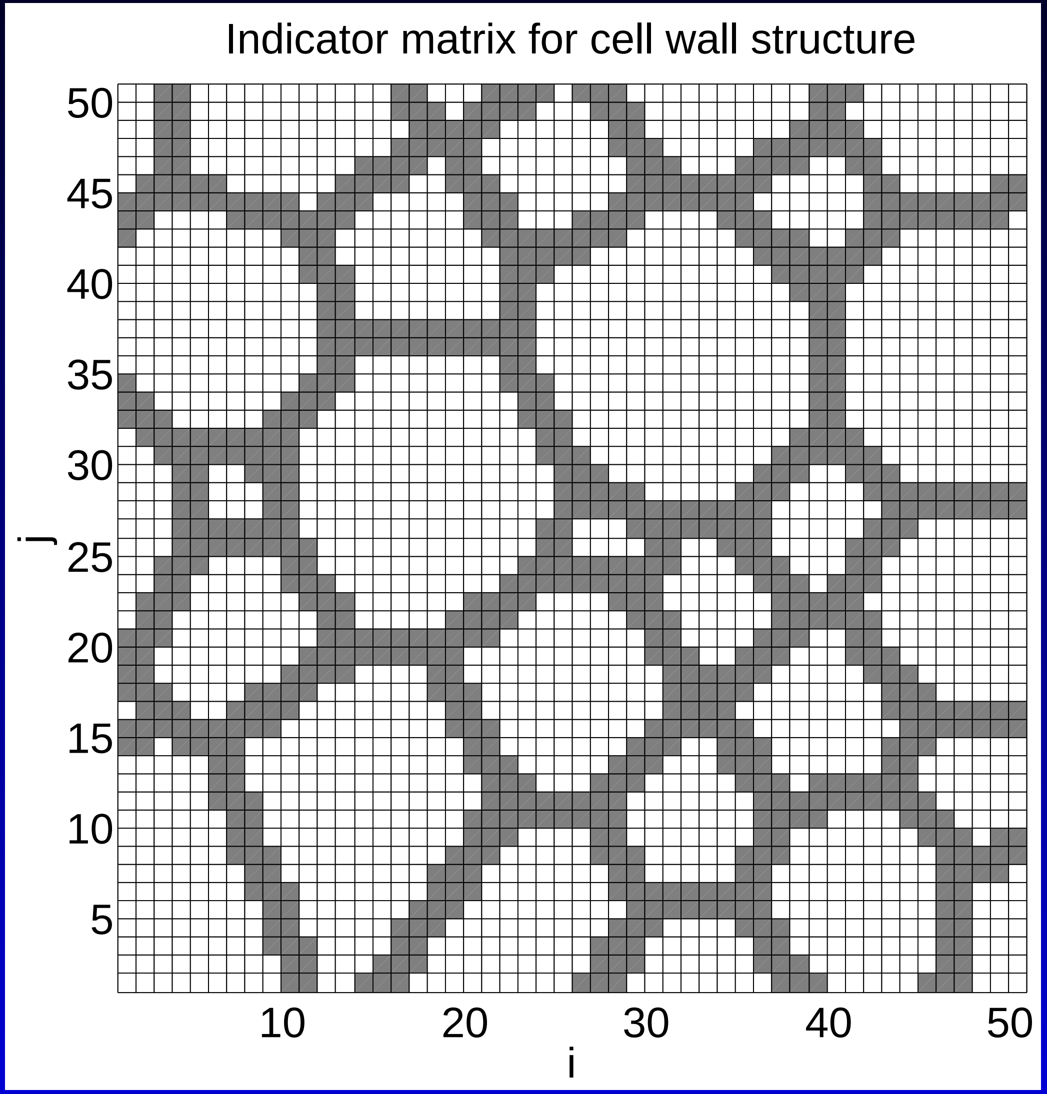
The “drop and roll” algorithm filled-in with disks of diameter at least the mean value minus two standard deviations (disks labeled 55 through 62 in decreasing order of size).

Truncated Domain



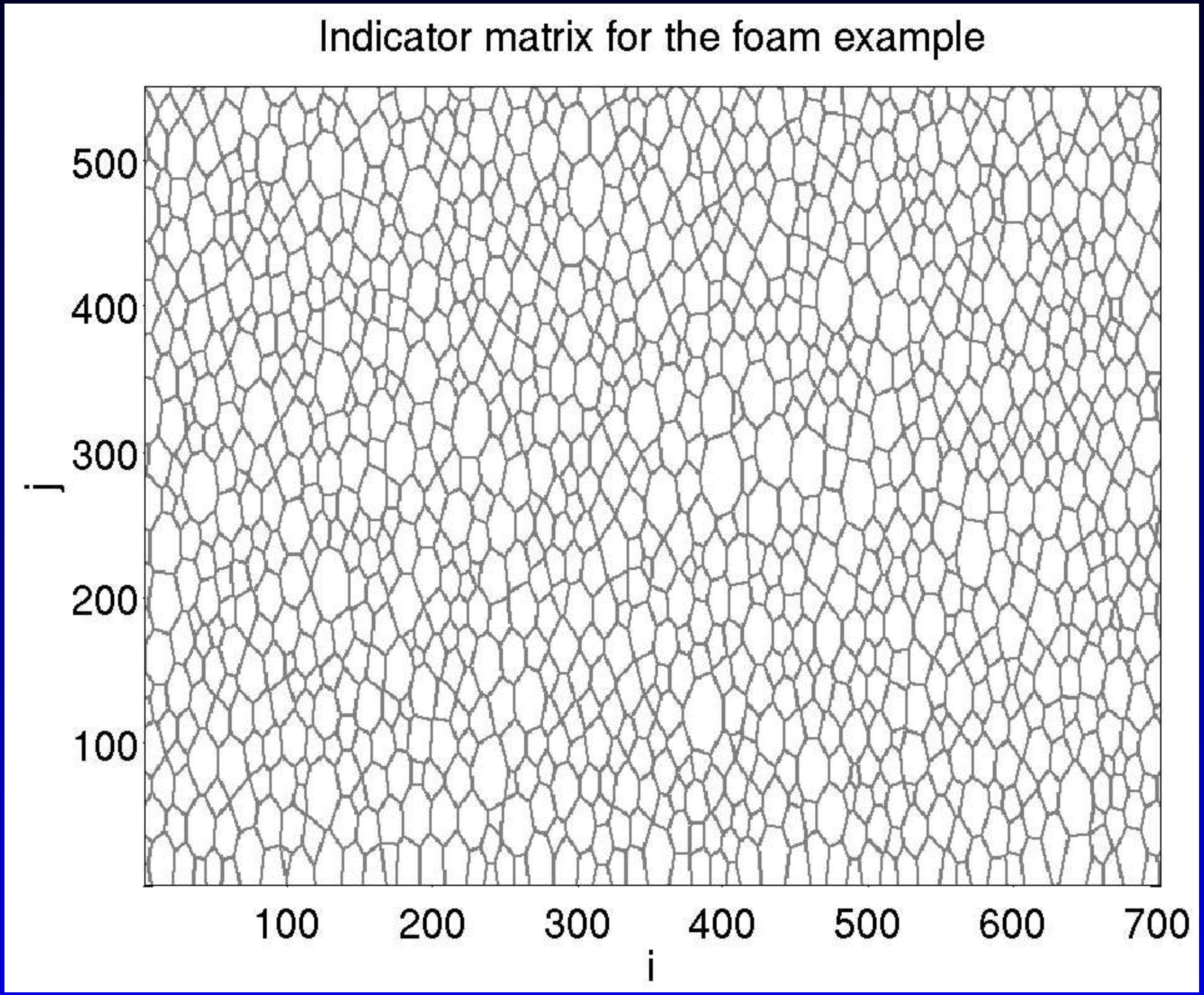
Apollonius graph on randomly close packed disks with a truncated domain indicated.

Indicator Matrix



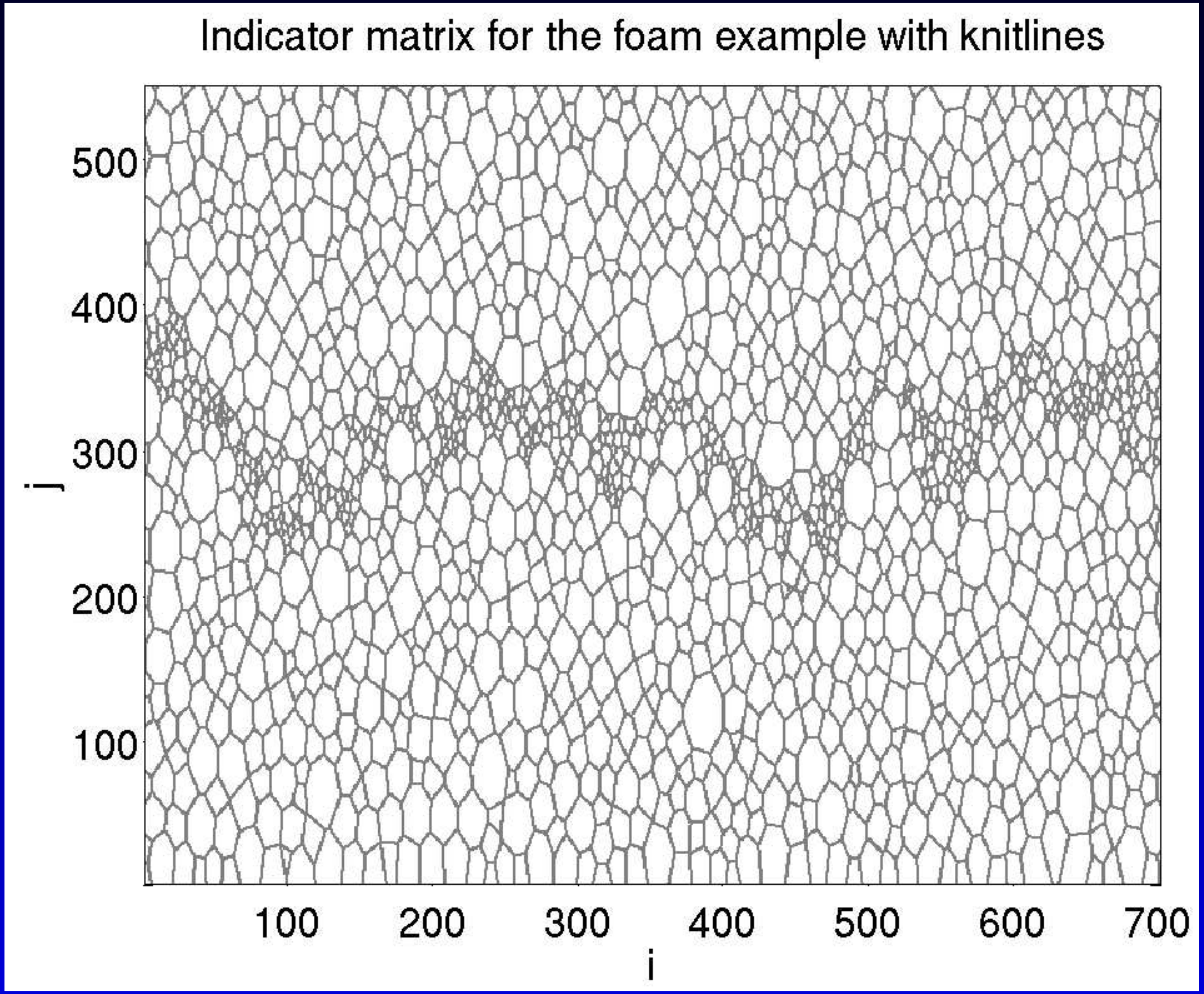
Indicator matrix for cell wall with thickness $2h$ where h is the meshsize.

Model of SOFI Microstructure



Apollonius graph is truncated and stretched, then edges are given a thickness and discretized to result in an indicator matrix as shown in the bitmap.

Model of SOFI with Knit Lines



Different sizes of drops are used to give different sized cells resulting in the appearance of knit lines.

Continuing Directions

- Modeling Approaches
  - Microscale scattering model
  - Match attenuation observed in data
- Computational Methods
  - Edge elements
  - ABC/PML
  - Faster time-marching
- Quantify Robustness wrt Uncertainty
  - Material properties
  - Geometry

Related Courses

- Linear Algebra
- ODEs
- PDEs
- Numerical Methods for the above
- Optimization
- Prob/Stat
- Interdisciplinary

Upcoming Conferences