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Trimming a Lévy Subordinator

Abstract:

Let $N = \sum_k \epsilon_{j_k}(\cdot)$ be PRM(ν), a Poisson random measure on $(0,\infty)$ with mean measure $\nu(\cdot)$. Suppose $\nu(\cdot)$ is finite in neighborhoods of ∞ with $Q(x) = \nu(x,\infty)$ as the finite tail function. If $\{\Gamma_l, l \geq 1\}$ are unit rate homogeneous Poisson points on $(0,\infty)$ and $\int_0^1 u\nu(du) < \infty$, we may generate a Lévy subordinator $X(t), t \geq 0$ and represent $X = X(1)$ as

$$X = \int_0^\infty uN(du) = \sum_{l=1}^\infty Q^{\leftarrow}(\Gamma_l),$$

a sum of Poisson jumps written in decreasing order. We may peel or trim off the r largest points and define

$${}^{(r)}X = \sum_{l=r+1}^\infty Q^{\leftarrow}(\Gamma_l), \quad Y^{(r)} = Q^{\leftarrow}(\Gamma_r)$$

giving the trimmed Levy random variable and the rth largest jump.

As $r \to \infty$, when does

$$({}^{(r)}X, Y^{(r)})$$

have a limit distribution (with appropriate centering and scaling)? Since it is always true that

$$\frac{{}^{(r)}X - \mu(Y^{(r)})}{\sigma(Y^{(r)})} = \frac{{}^{(r)}X - \int_0^{Y^{(r)}} u\nu(du)}{\int_0^{Y^{(r)}} u^2\nu(du)} \Rightarrow N_X = N(0,1)$$

(since $r \to \infty$ means we mash down the size of the jumps), the answer differs if centerings are allowed to be random or not. With deterministic centerings, extended regular variation must be employed to get a solution.

This talk is based on joint work with Ross Maller, Boris Buchmann, Yugang Ipsen, Australian National University.

References:

[1] B. Buchmann, R. Maller, and S. Resnick. Processes of rth Largest. *ArXiv e-prints*, July 2016. `http://adsabs.harvard.edu/abs/2016arXiv160708674B`. Submitted: Extremes.

[2] Y. Ipsen, R. Maller and S. Resnick. Ratios of Ordered Points of Point Processes with Regularly Varying Intensity Measures. In preparation.

[3] Y. Ipsen, R. Maller and S. Resnick. Joint limit behavior of trimmed subordinators and the rth largest jump. Forthcoming.